기초과학연구원
Institute for Basic Science

On gauge-dependence of gravitational waves from 1st-order phase transitions

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based on

Cheng-Wei Chiang (Natl Taiwan U), E.S., arXiv: 1707.06765 (PLB)

Outline

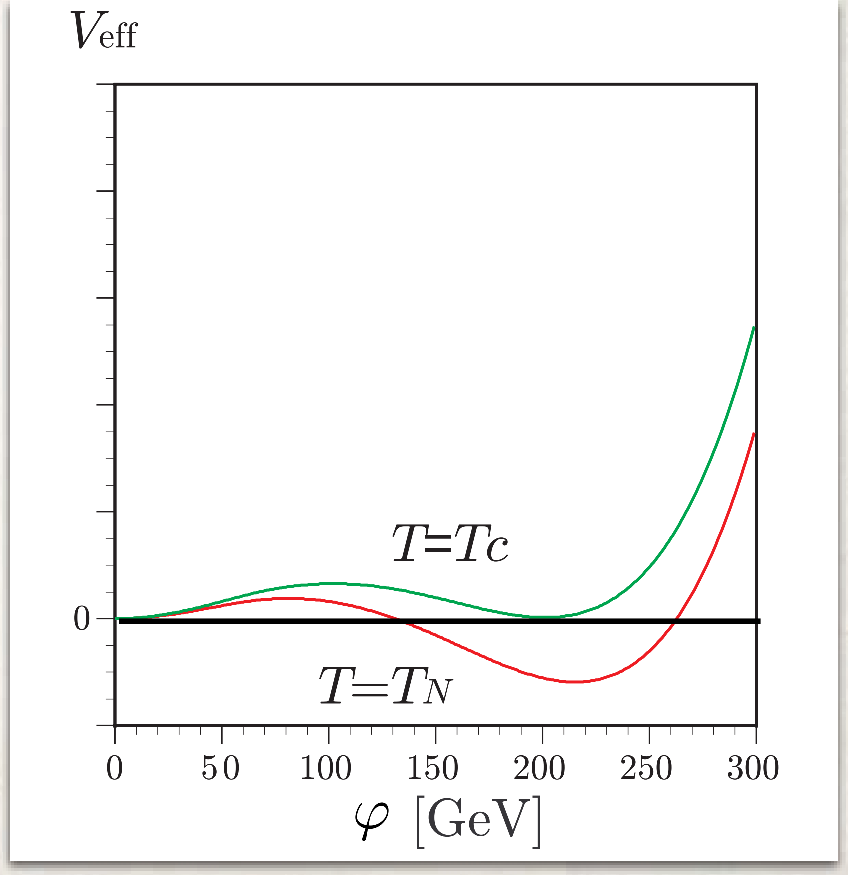
- Introduction
    - Gauge-dependence (ξ) of the effective potential
- Impact of ξ on 1^{st}-order phase transition in classical scale-inv. U(1) models: T_N, GW
- Summary

Introduction

- 1^{st}-order phase transition (PT) has interesting physical implications:
  Electroweak Baryogenesis, Gravitational Waves (GW), etc.

- Mostly, effective potential is used for such calculations.

1st-order PT



problem

- Effective potential inherently depends on gauge-fixing parameter (ξ).

- Nucleation temperature (T_N), GW can be ξ dependent.

Q. How (numerically) serious?

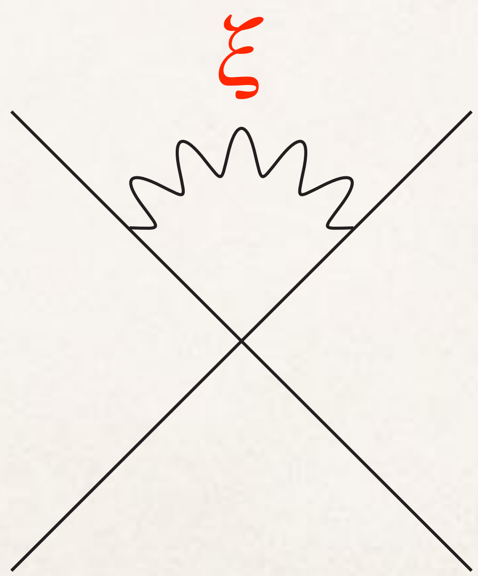
Thorny problem

Effective potential is gauge dependent!!

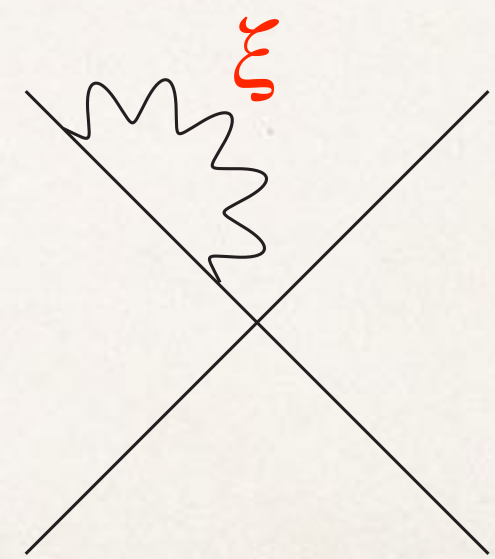
Jackiw, PRD9,1686 (1974)

Because

$V_{eff} \ni$



$\not\ni$



1PI diagrams only

Leg corrections are needed to remove the ξ dependence.

Gauge dependence of V_{eff}

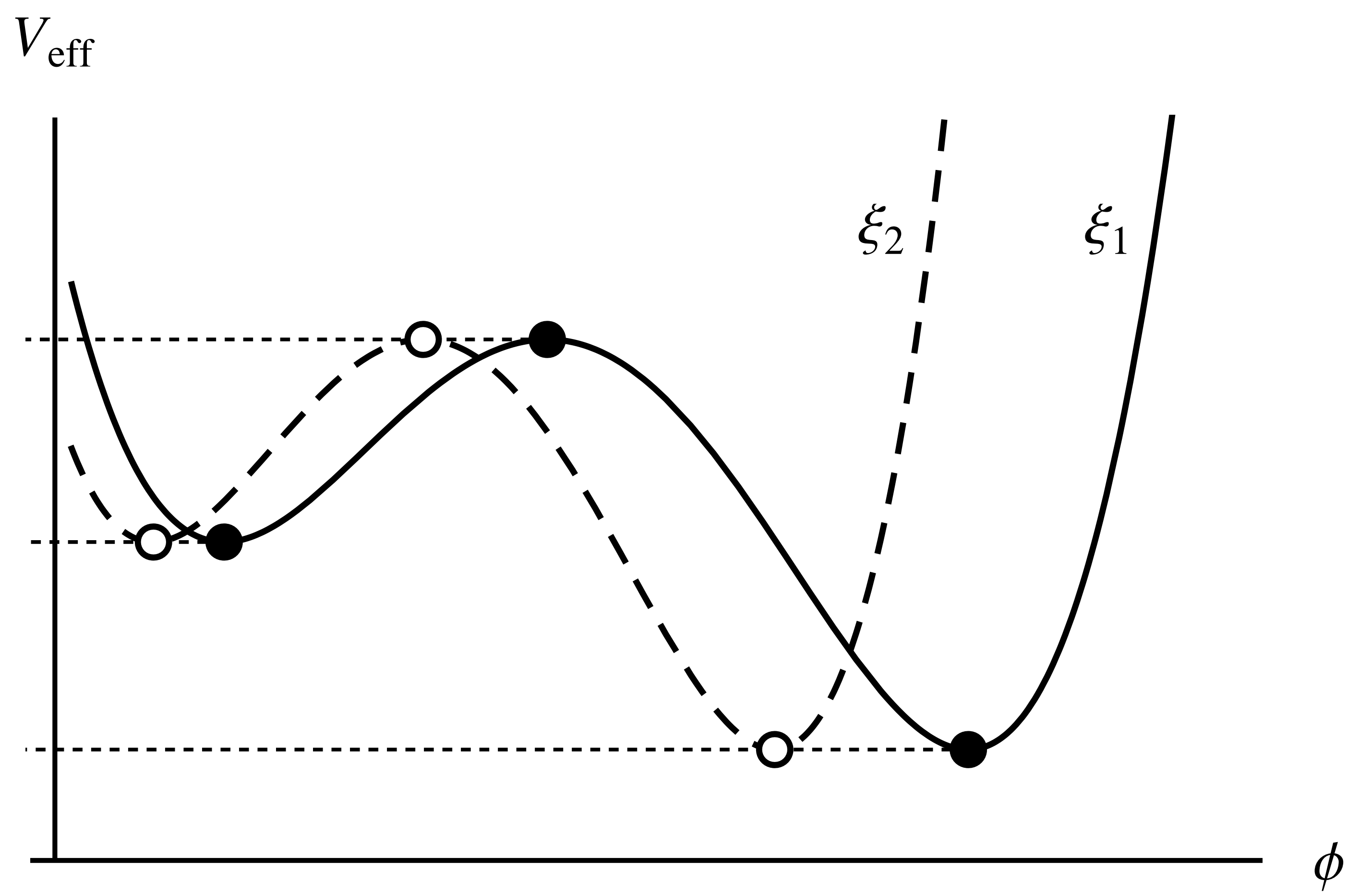


Fig. taken from H. Patel and M. Ramsey-Musolf, JHEP,07(2011)029

- Generally, VEV depends on a gauge parameter ξ
- Energies at stationary points do not depend on ξ

$$\frac{\partial V_{\rm eff}}{\partial \xi} = C(\varphi, \xi)\frac{\partial V_{\rm eff}}{\partial \varphi}$$

(Nielsen-Fukuda-Kugo (NFK) identity)

Abelian-Higgs model

~ an example ~

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu\Phi|^2 - V(|\Phi|^2),$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad D_\mu\Phi = (\partial_\mu - ieA_\mu)\Phi,$$

$$V(|\Phi|^2) = -\nu^2|\Phi|^2 + \frac{\lambda}{4}|\Phi|^4, \quad \Phi(x) = \frac{1}{\sqrt{2}}\big(v + h(x) + iG(x)\big).$$

gauge boson: $$D^{-1}_{\mu\nu}(k) = (-k^2 + \bar{m}_A^2)\Pi^T_{\mu\nu}(k) + \frac{1}{\xi}(-k^2 + \xi\bar{m}_A^2)\Pi^L_{\mu\nu}(k),$$

$$\Pi^T_{\mu\nu}(k) = \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right), \quad \Pi^L_{\mu\nu}(k) = \frac{k_\mu k_\nu}{k^2},$$

NG boson: $$\Delta_G^{-1}(k) = k^2 - \bar{m}_G^2 - \xi\bar{m}_A^2$$

ghost: $$\Delta_c^{-1}(k) = i(k^2 - \xi\bar{m}_A^2)$$

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1-loop effective potential

e.g., Abelian-Higgs model

$$\mu^\epsilon V_1^{A+G+c}(\varphi,\xi) = -\frac{i}{2}\mu^\epsilon \int \frac{d^Dk}{(2\pi)^D}\Big[(D-1)\ln(-k^2+\bar{m}_A^2)+\ln(-k^2+\xi\bar{m}_A^2) + \ln(-k^2+\xi\bar{m}_A^2)+\ln\left(1+\frac{\bar{m}_G^2}{-k^2+\xi\bar{m}_A^2}\right) - 2\ln(-k^2+\xi\bar{m}_A^2)\Big]$$

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ξ -dependence disappears at $\bar{m}_G^2(\varphi=v)=0, \quad \left.\frac{\partial V_0}{\partial\varphi}\right|_{\varphi=v}=0$

NFK identity at 1-loop level:

$$\frac{\partial V_1(\varphi,\xi)}{\partial\xi} = C(\varphi,\xi)\frac{\partial V_0(\varphi)}{\partial\varphi}$$

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NFK identity at 1-loop level:

1-loop

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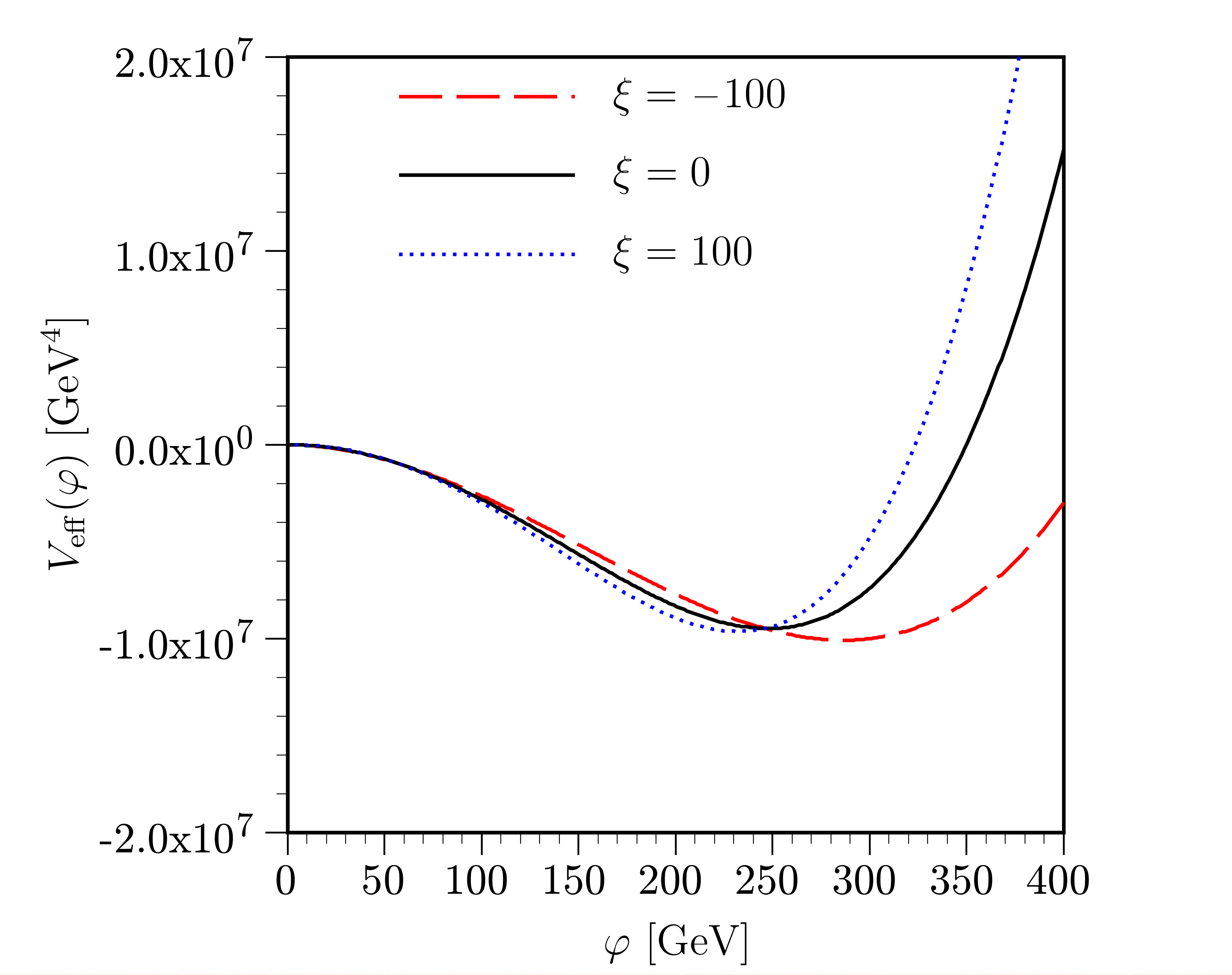
NFK identity at 1-loop level:

1-loop Tree

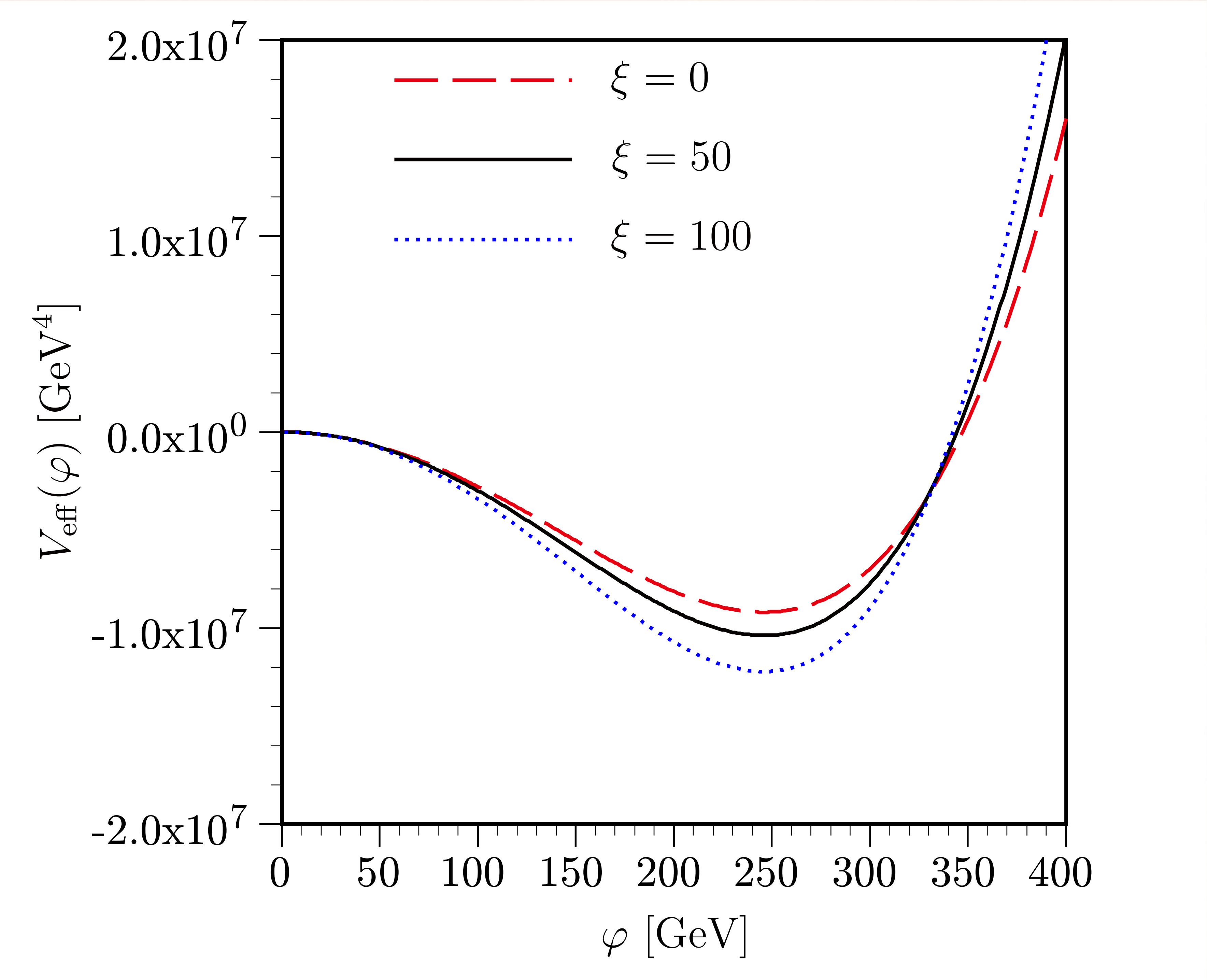
$$\frac{\partial V_1(\varphi, \xi)}{\partial \xi} = C(\varphi, \xi) \frac{\partial V_0(\varphi)}{\partial \varphi}$$

Plotting $V_{eff}=V_0+V_1$,



No ξ -dependence at $\dfrac{\partial V_0}{\partial \varphi} = 0$ but it is no longer a minimum at 1-loop level.

When 1-loop minimization condition is imposed, $\frac{\partial(V_0+V_1)}{\partial\varphi}=0$



Energy at ϕ =246 GeV depends on ξ !!

1-loop effective potential at T≠0

$$V_1(\varphi, \xi; T) = \sum_i \frac{T^4}{2\pi^2} I_B(a_i^2),$$

where

$$I_B(a^2) = \int_0^\infty dx \; x^2 \ln\left[1 - e^{-\sqrt{x^2+a^2}}\right].$$

Using a high-T expansion of $I_B(a^2)$, one gets

$$\begin{aligned}
&V_1(\varphi, \xi) + V_1(\varphi, \xi; T) \\
&= \frac{T^2}{24}(\bar{m}_h^2 + \bar{m}_G^2 + 3\bar{m}_A^2) - \frac{T}{12\pi}\left[(\bar{m}_h^2)^{3/2} + (\bar{m}_G^2 + \xi\bar{m}_A^2)^{3/2} + (3 - \xi^{3/2})(\bar{m}_A^2)^{3/2}\right] \\
&\quad + \frac{1}{64\pi^2}\left[\bar{m}_h^4 \ln\frac{\alpha_B T^2}{\bar{\mu}^2} + (\bar{m}_G^2 + \xi\bar{m}_A^2)^2 \ln\frac{\alpha_B T^2}{\bar{\mu}^2} + 3\bar{m}_A^4\left(\ln\frac{\alpha_B T^2}{\bar{\mu}^2} + \frac{2}{3}\right) - (\xi\bar{m}_A^2)^2 \ln\frac{\alpha_B T^2}{\bar{\mu}^2}\right].
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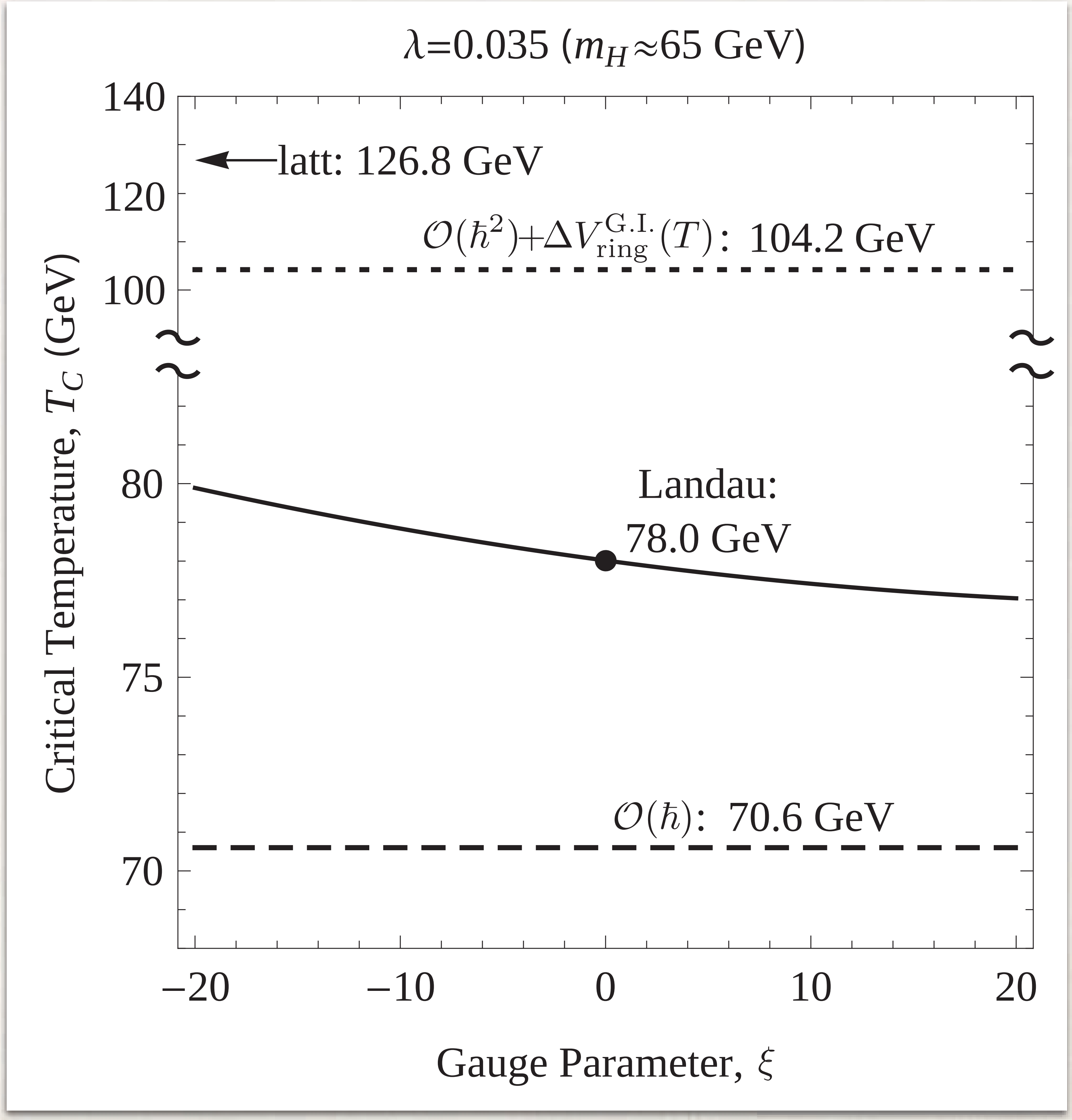
V_{eff} at T≠0 also depends on ξ except "T^2-terms".

SM case

[H.Patel, M.Ramsey-Musolf, JHEP,07(2011)029]



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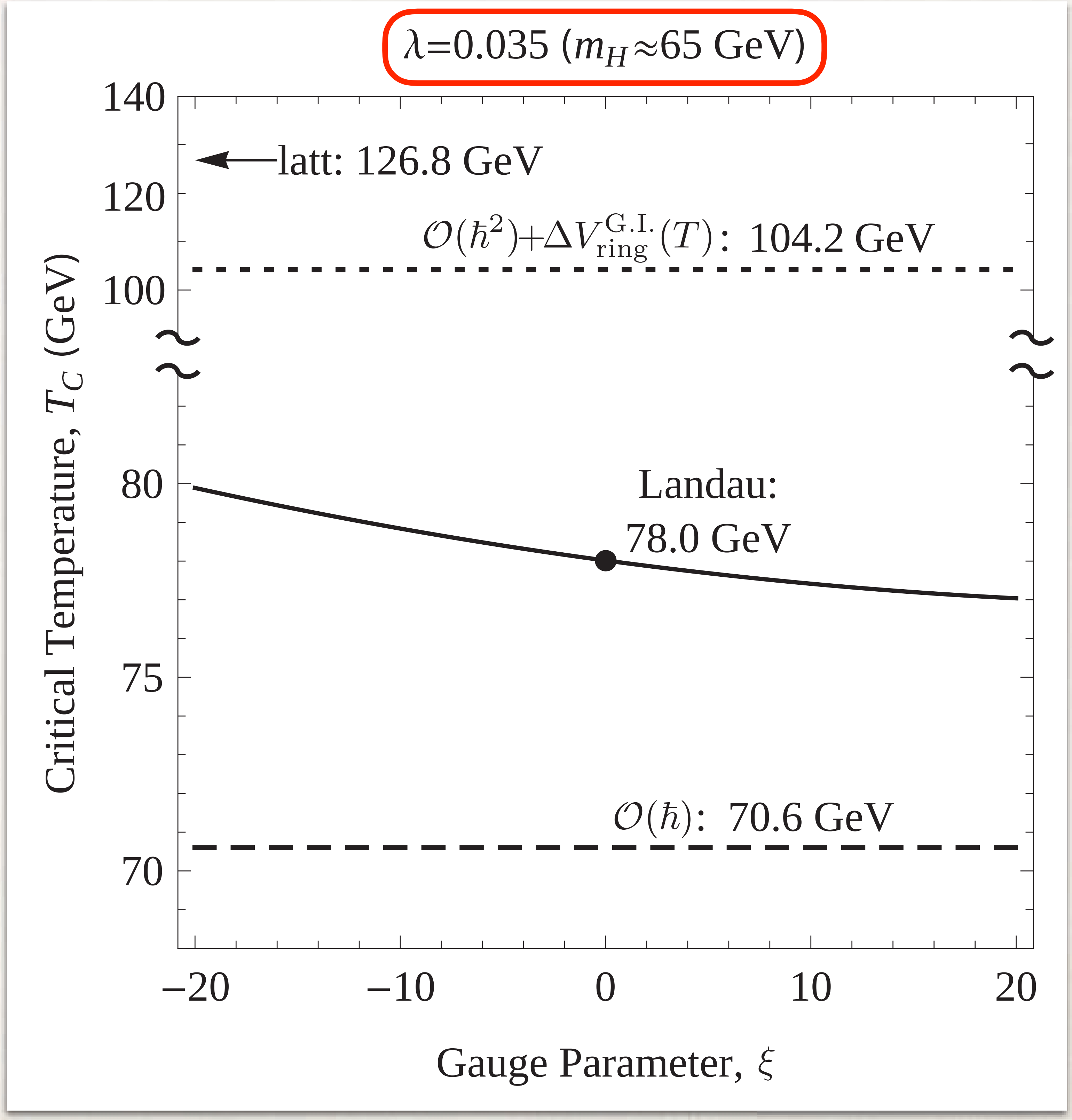

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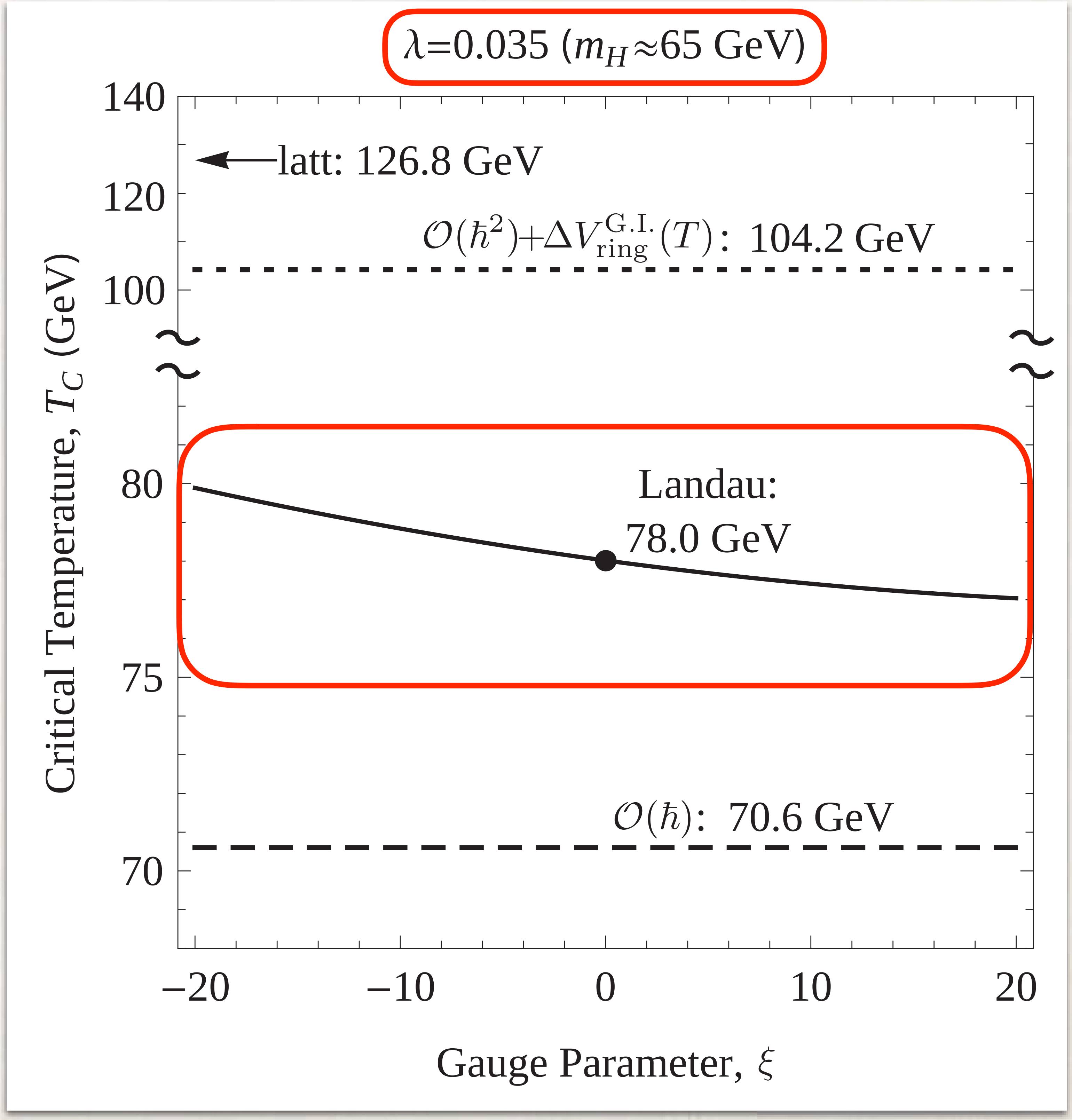

Classical scale-inv. U(1) model

SM + U(1)′ w/ scale symmetry

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}'} - \frac{1}{4} Z'_{\mu\nu} Z'^{\mu\nu} + |D_\mu S|^2 - V(H,S)$$

$$Z'_{\mu\nu} = \partial_\mu Z'_\nu - \partial_\nu Z'_\mu, \; D_\mu S = (\partial_\mu + i g' Q'_S Z'_\mu) S,$$

scalar potential

$$V(H,S) = \lambda_H (H^\dagger H)^2 + \lambda_{HS} H^\dagger H |S|^2 + \lambda_S |S|^4$$

singlet scalar field: $S(x) = \frac{1}{\sqrt{2}} \left(v_S + h_S(x) + i G(x) \right)$

After U(1) is radiatively broken (<S>≠0), EW symmetry is broken if λ_{HS} < 0. $m_h^2 = -\lambda_{HS} v_S^2 \longrightarrow -\lambda_{HS} = m_h^2 / v_S^2 = \mathcal{O}(10^{-3})$

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Classical scale-inv. U(1) model

ξ dependence is different from the massive U(1) model case.

$$V_{\rm eff}(\varphi_S) = \frac{\lambda_S}{4}\varphi_S^4 + 3\frac{\bar{m}_{Z'}^4}{64\pi^2}\left(\ln\frac{\bar{m}_{Z'}^2}{\bar{\mu}^2} - \frac{5}{6}\right) + \frac{\bar{m}_{G,\xi}^4}{64\pi^2}\left(\ln\frac{\bar{m}_{G,\xi}^2}{\bar{\mu}^2} - \frac{3}{2}\right) - \frac{(\xi\bar{m}_{Z'}^2)^2}{64\pi^2}\left(\ln\frac{\xi\bar{m}_{Z'}^2}{\bar{\mu}^2} - \frac{3}{2}\right),$$

where $\bar{m}_{Z'}^2 = (g'Q'_S\varphi_S)^2, \quad \bar{m}_{G,\xi}^2 = \lambda_S\varphi_S^2 + \xi\bar{m}_{Z'}^2.$

Minimization condition -> $\lambda_S = O(g'^4/16\pi^2)$

One gets

$$V_{\rm eff}(\varphi_S) \simeq \frac{3\bar{m}_{Z'}^4}{64\pi^2}\left(\ln\frac{\varphi_S^2}{v_S^2} - \frac{1}{2}\right),$$

ξ independent!!

- Finite-T 1-loop effective potential is also ξ independent.
- ξ dependence will appear from 2-loop order.

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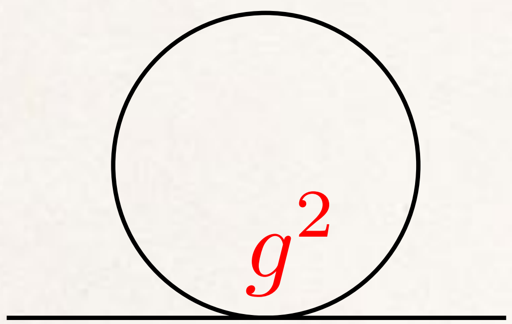
Thermal resummation

At high T



$\sim g^2 T^2$

<u>**Prescription**</u>

e.g. NG boson

$$\bar{m}^2_{G,\xi} \to \bar{m}^2_{G,\xi} + \Delta m_S^2$$

to leading order: $\Delta m_S^2 = \dfrac{(g' Q'_S)^2}{4} T^2$

$$V_{\text{eff}}(\varphi_S;T) \underset{\xi\text{-part}}{=} \frac{(\xi \bar{m}^2_{Z'} + \Delta m_S^2)^2}{64\pi^2}\left(\ln\frac{\xi \bar{m}^2_{Z'} + \Delta m_S^2}{\bar{\mu}^2} - \frac{3}{2}\right) - \frac{(\xi \bar{m}^2_{Z'})^2}{64\pi^2}\left(\ln\frac{\xi \bar{m}^2_{Z'}}{\bar{\mu}^2} - \frac{3}{2}\right) + \frac{T^4}{2\pi^2}\left[I_B\left(\frac{\xi \bar{m}^2_{Z'} + \Delta m_S^2}{T^2}\right) - I_B\left(\frac{\xi \bar{m}^2_{Z'}}{T^2}\right)\right].$$

where I_B is a 1-loop thermal function.

V_{eff} is no longer ξ independent due to $\Delta m_S^2 \neq 0$

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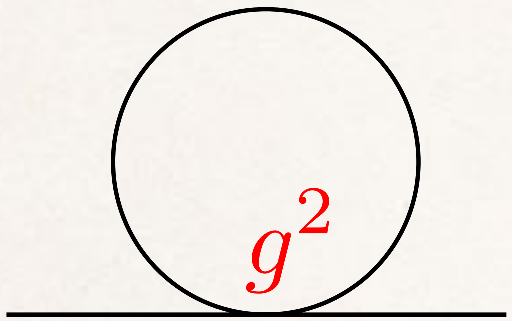

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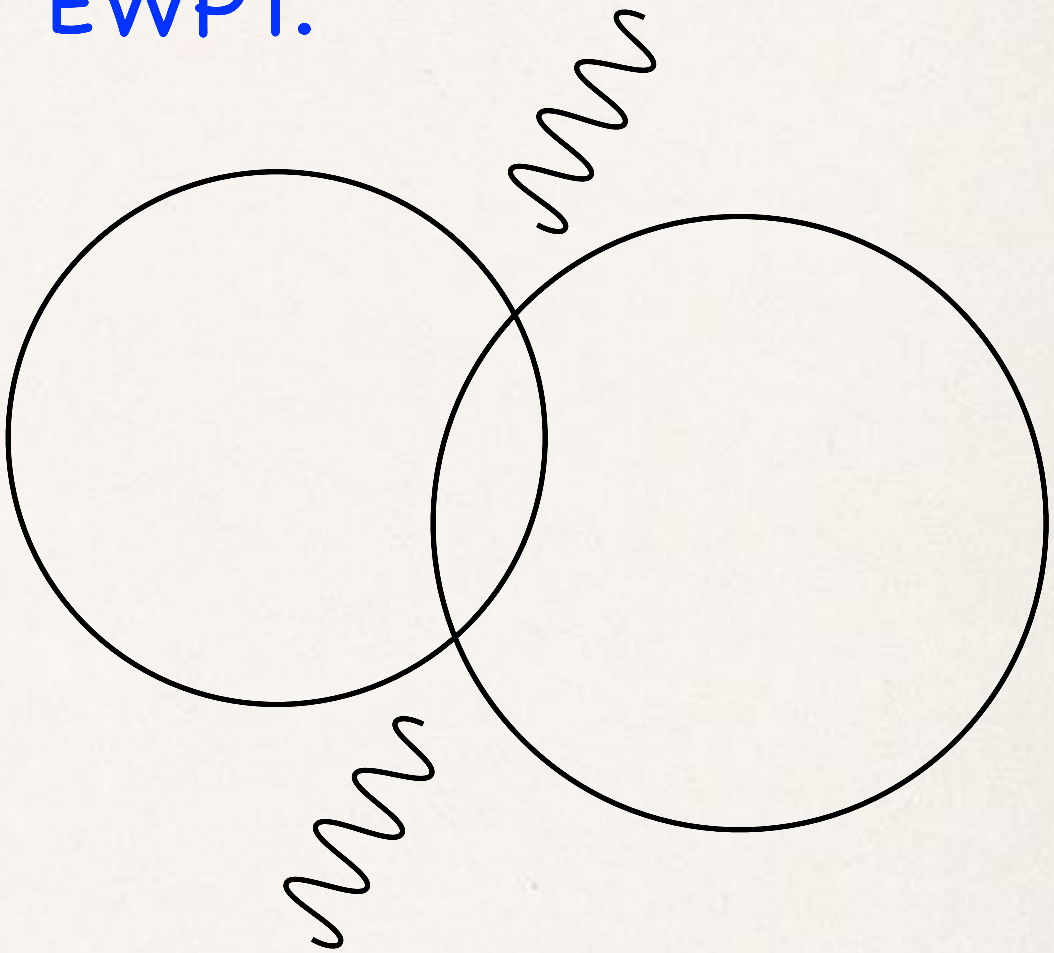
Gravitational Waves from 1st-order EWPT

GWs are induced by the 1st-order EWPT.

Sources of GW

(1) Bubble collisions,
(2) Sound waves,
(3) Turbulence



See Ref. [C.Caprini et al, 1512.06239(JCAP)]

2 important parameters: [Grojean, Servant, hep-ph/0607107(PRD)]

latent heat (α) , duration of PT (β)

$$\alpha \equiv \frac{\epsilon(T_*)}{\rho_{\rm rad}(T_*)} \quad \text{and} \quad \beta \equiv H_* T_* \frac{d}{dT}\left(\frac{S_3(T)}{T}\right)\Bigg|_{T=T_*}, \quad \epsilon(T) = \Delta V_{\rm eff} - T\frac{\partial \Delta V_{\rm eff}}{\partial T} \quad \text{and} \quad \rho_{\rm rad}(T) = \frac{\pi^2}{30} g_*(T) T^4,$$

Gravitational Waves from 1st-order EWPT

[C.Caprini et al, 1512.06239(JCAP)]

$$\Omega_{\rm GW}h^2 = \Omega_{\rm col}h^2 + \Omega_{\rm sw}h^2 + \Omega_{\rm turb}h^2$$

Dominant source is sound waves:

$$\Omega_{\rm sw}h^2(f) = 2.65\times 10^{-6}\tilde{\beta}^{-1}\left(\frac{\kappa_v\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{1/3} v_w \left(\frac{f}{f_{\rm sw}}\right)^3\left(\frac{7}{4+3(f/f_{\rm sw})^2}\right)^{7/2},$$

$$f_{\rm sw} = 1.9\times 10^{-2}\ {\rm mHz}\ \frac{\tilde{\beta}}{v_w}\left(\frac{T_*}{100\ {\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{1/6}, \qquad \tilde{\beta} = \frac{\beta}{H_*}$$

$$\kappa_v \simeq \alpha/(0.73+0.083\sqrt{\alpha}+\alpha)\ \text{for}\ v_w \simeq 1.$$

- Most calculations of α & β in the literature depends on ξ.
- How much ξ dependence can affect GW?

Impact of ξ on T_N

[Cheng-Wei Chiang, E.S., 1707.06765 (PLB)]

$$Q'_S = 2,\ \alpha' = g'^2/4\pi = 0.015,\ m_{Z'} = 4.5 \text{ TeV and } m_{\nu_{R1,2,3}} = 1.0 \text{ TeV}.$$

$$S_3 = 4\pi \int_0^\infty dr\ r^2 \left[\frac{1}{2}\left(\frac{d\phi_S}{dr}\right)^2 + V_{\text{eff}}(\phi_S; T)\right]$$

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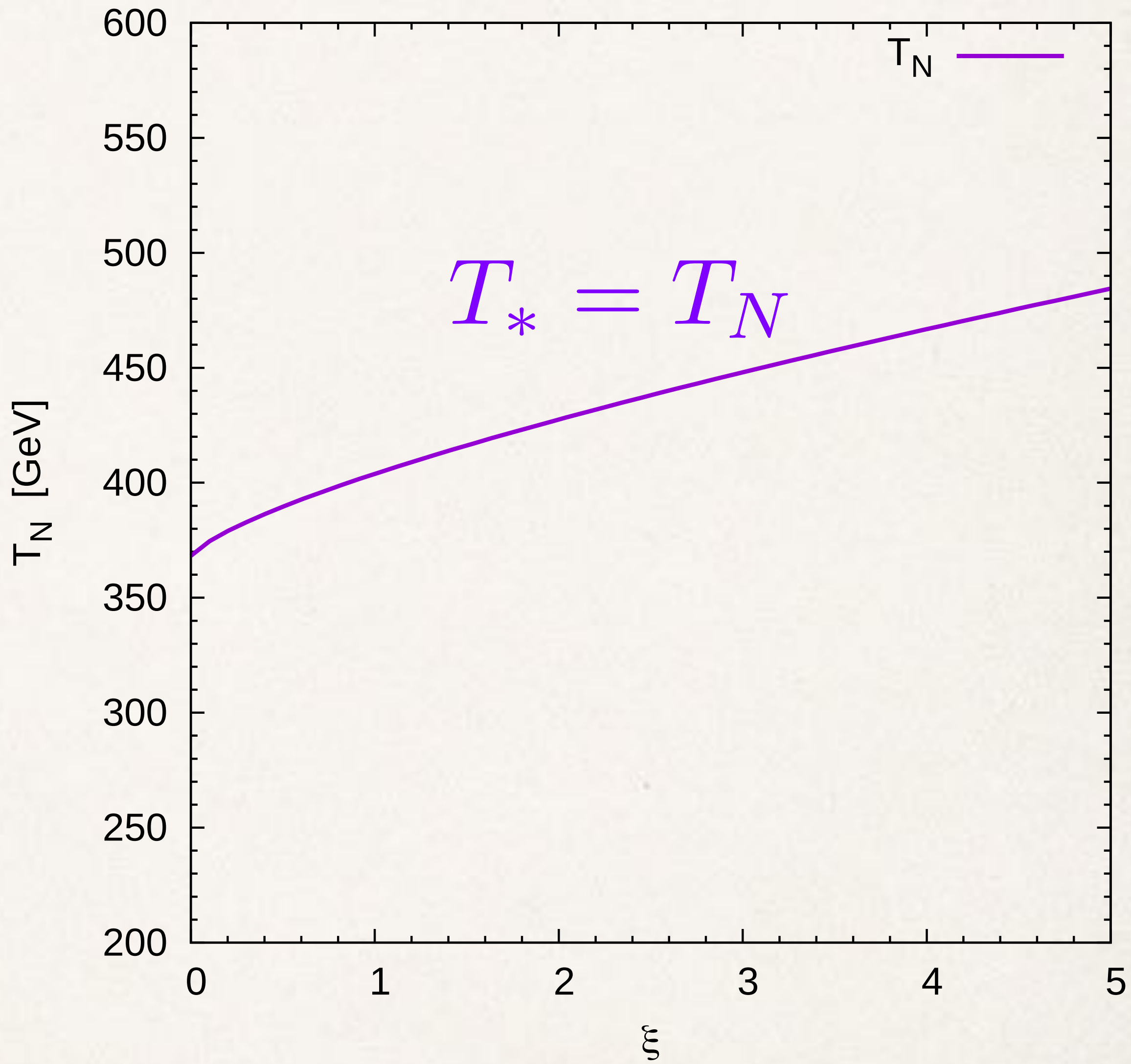

	no resum	$\xi = 0$	$\xi = 1$	$\xi = 5$
$v_S(T_*)/T_*$	$5.181/0.328 = 15.8$	$5.181/0.368 = 14.1$	$5.180/0.405 = 12.8$	$5.163/0.490 = 10.5$
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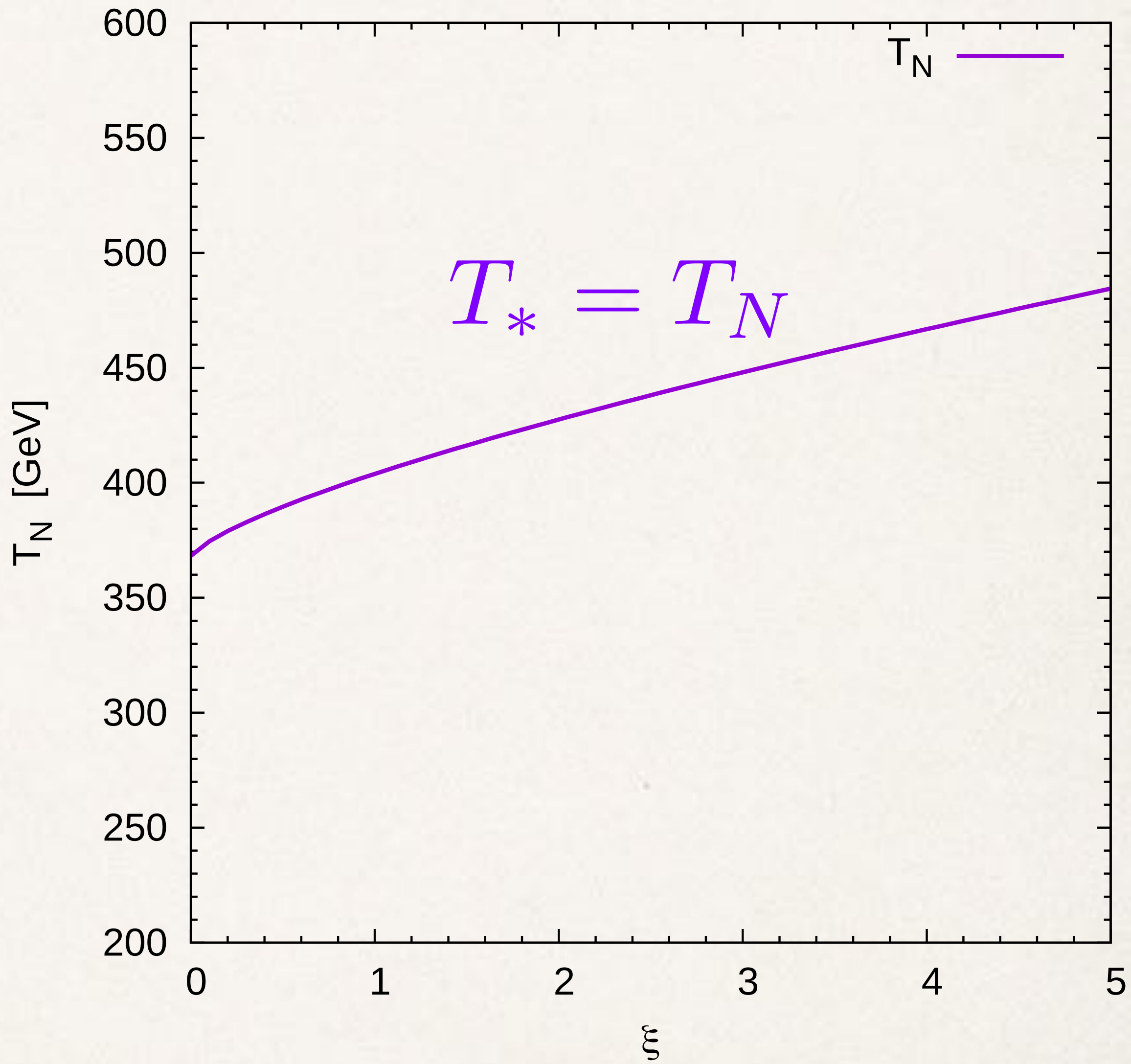

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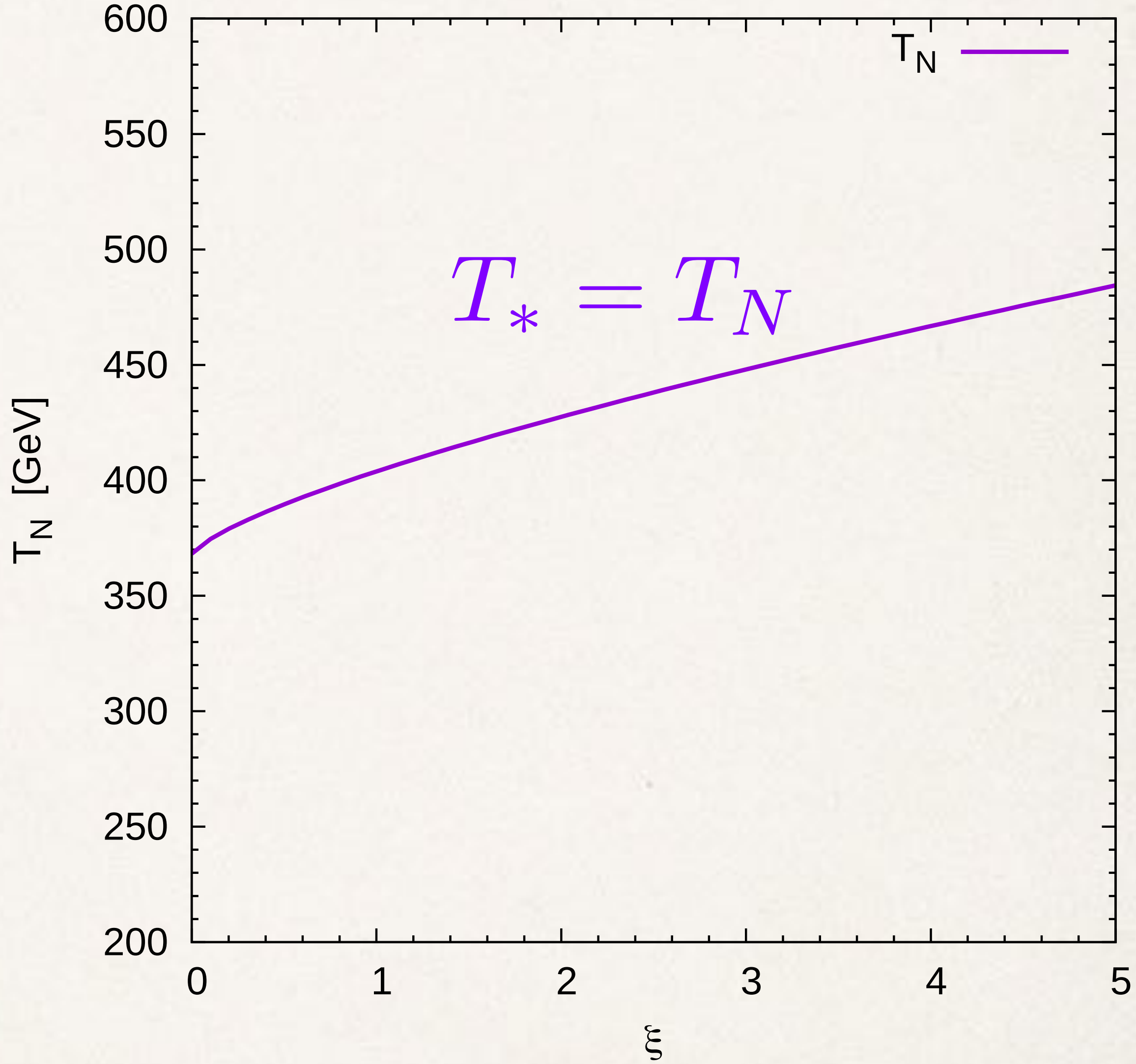

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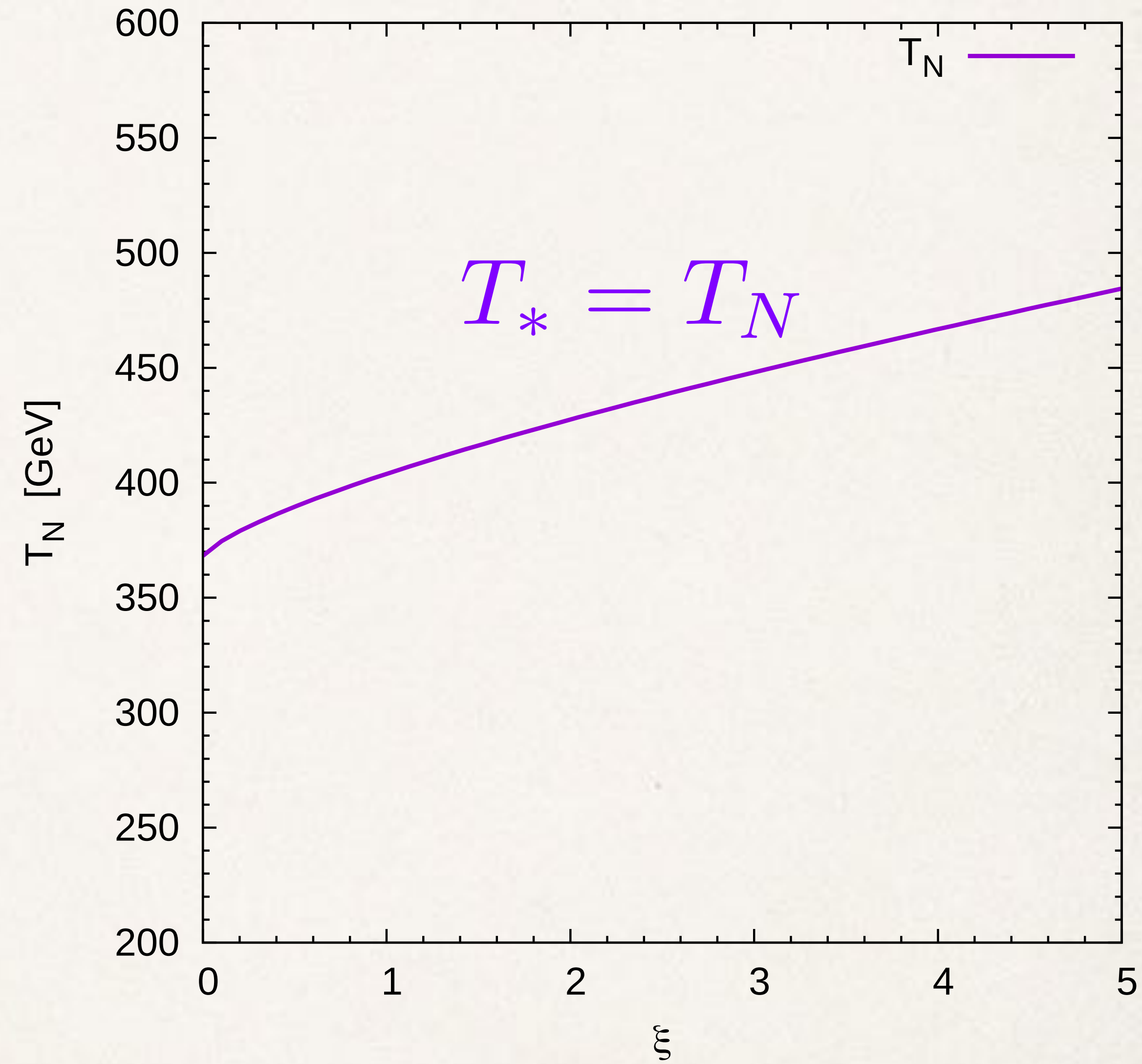

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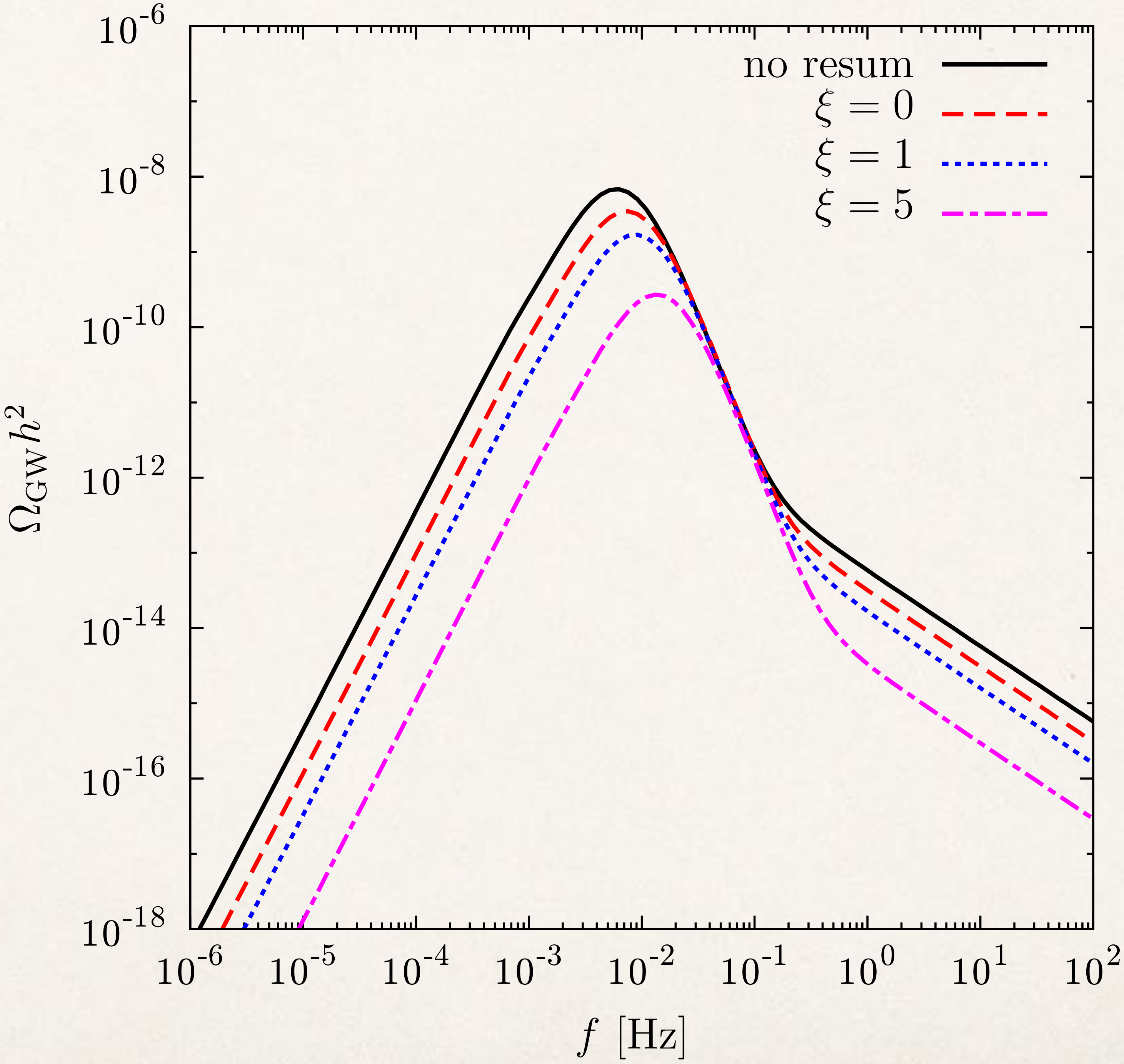

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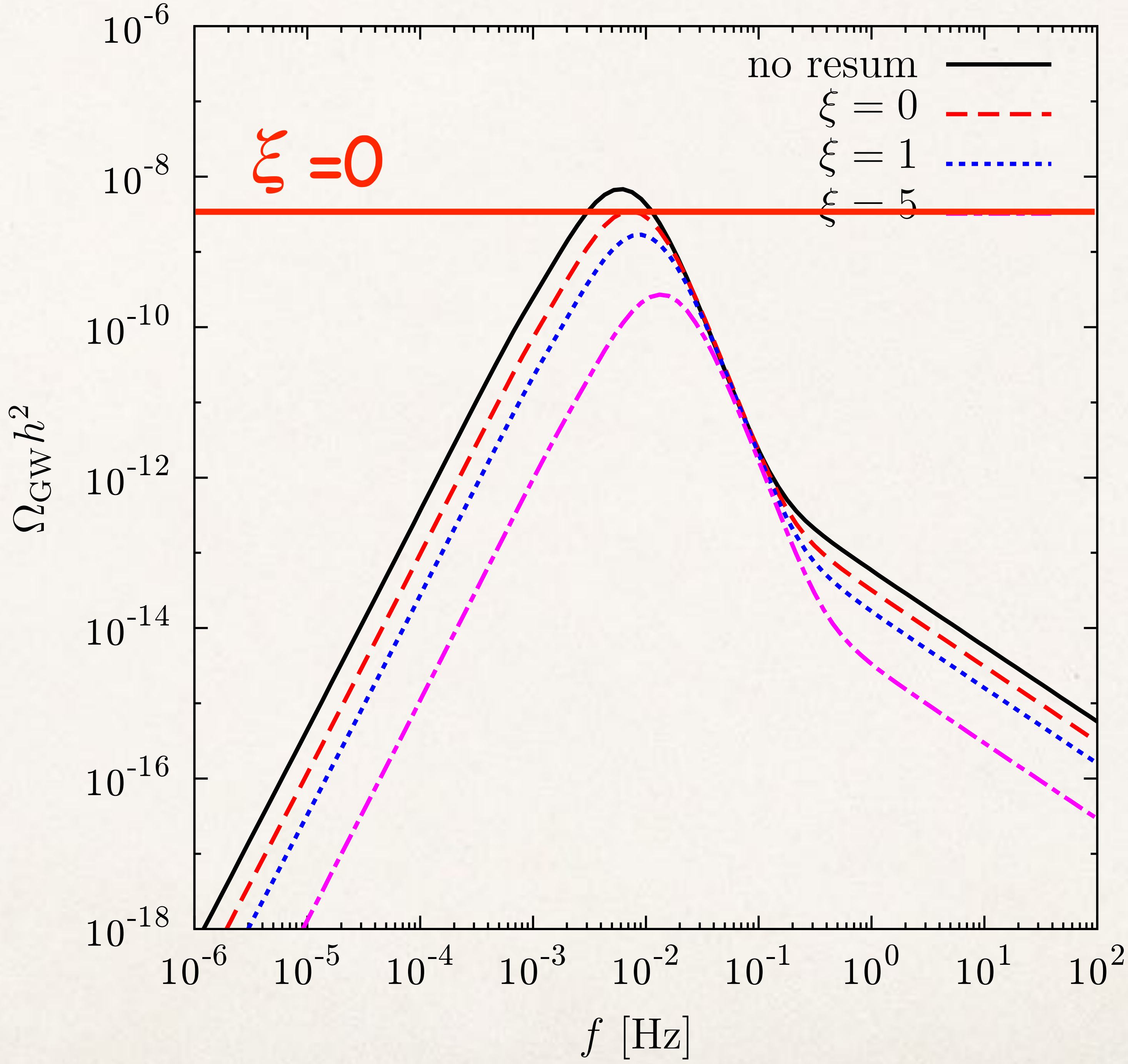

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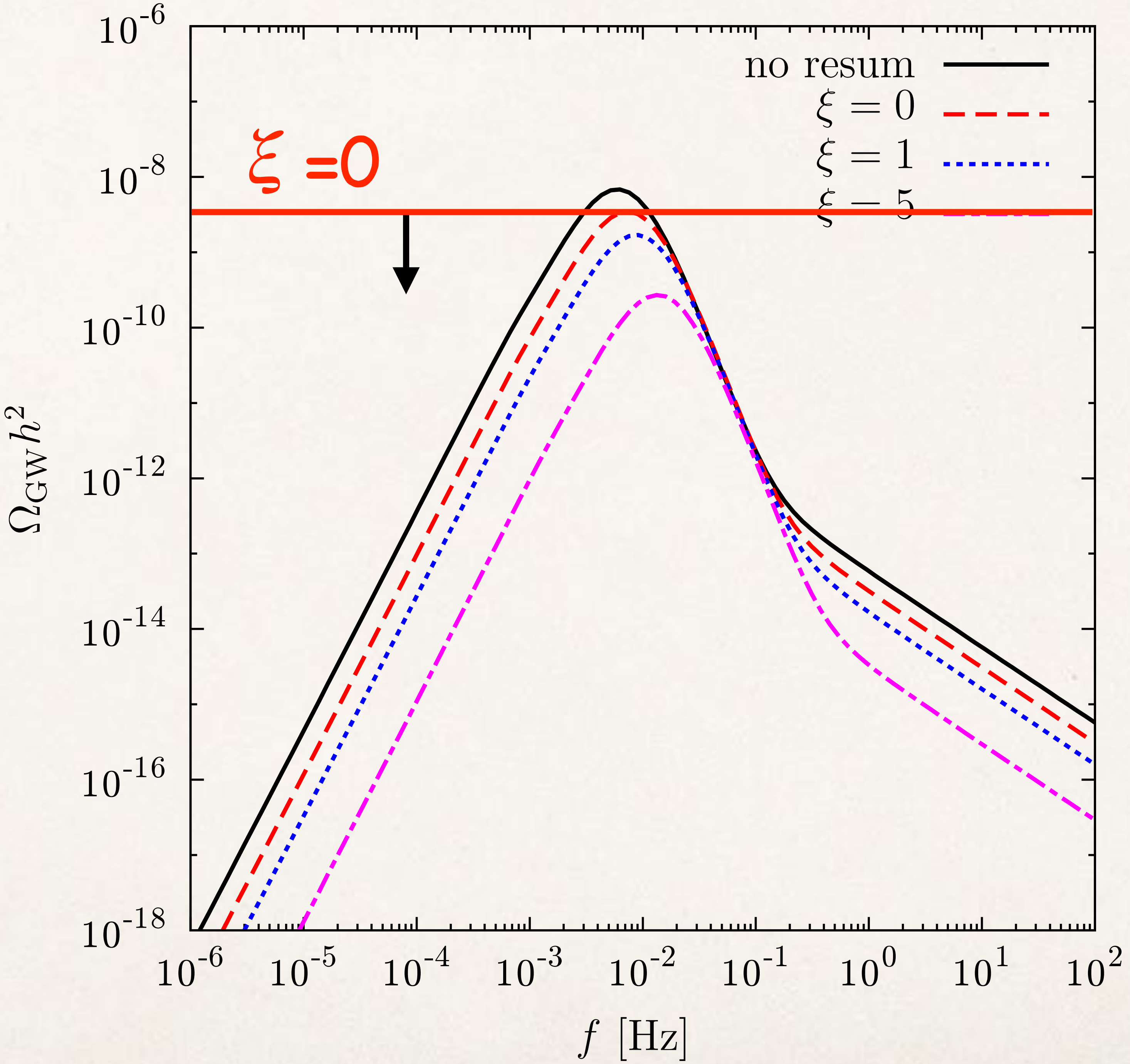

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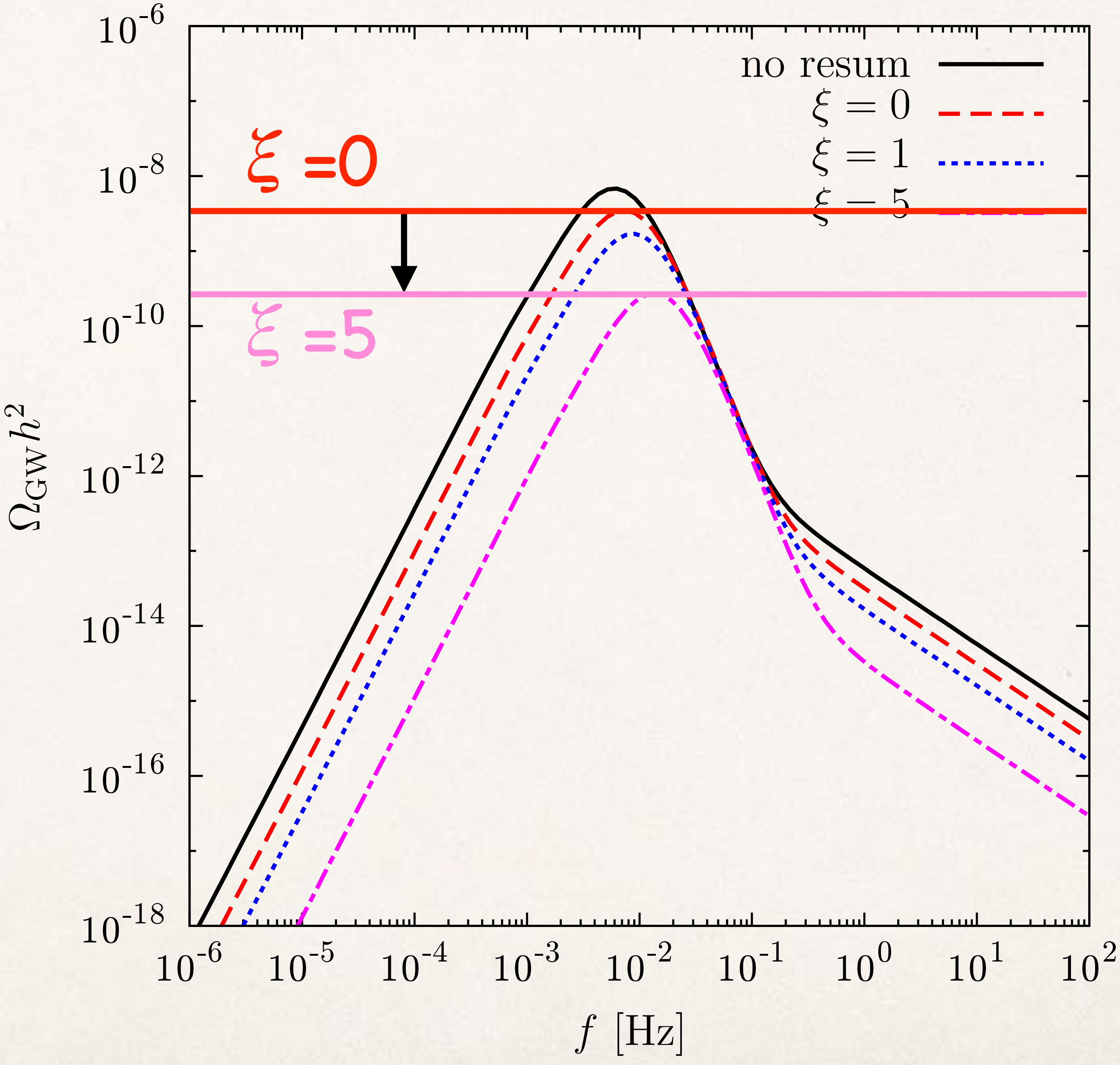

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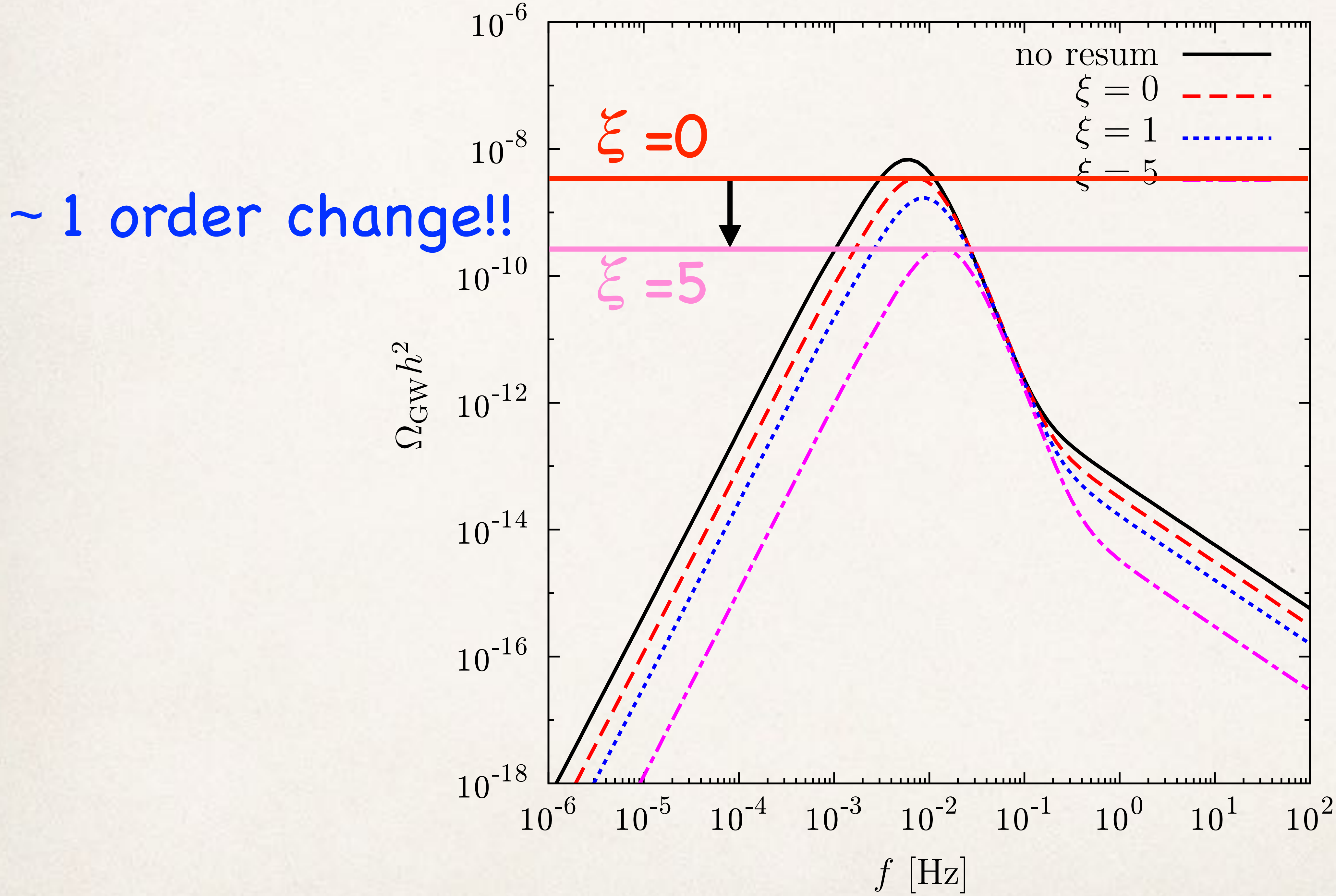
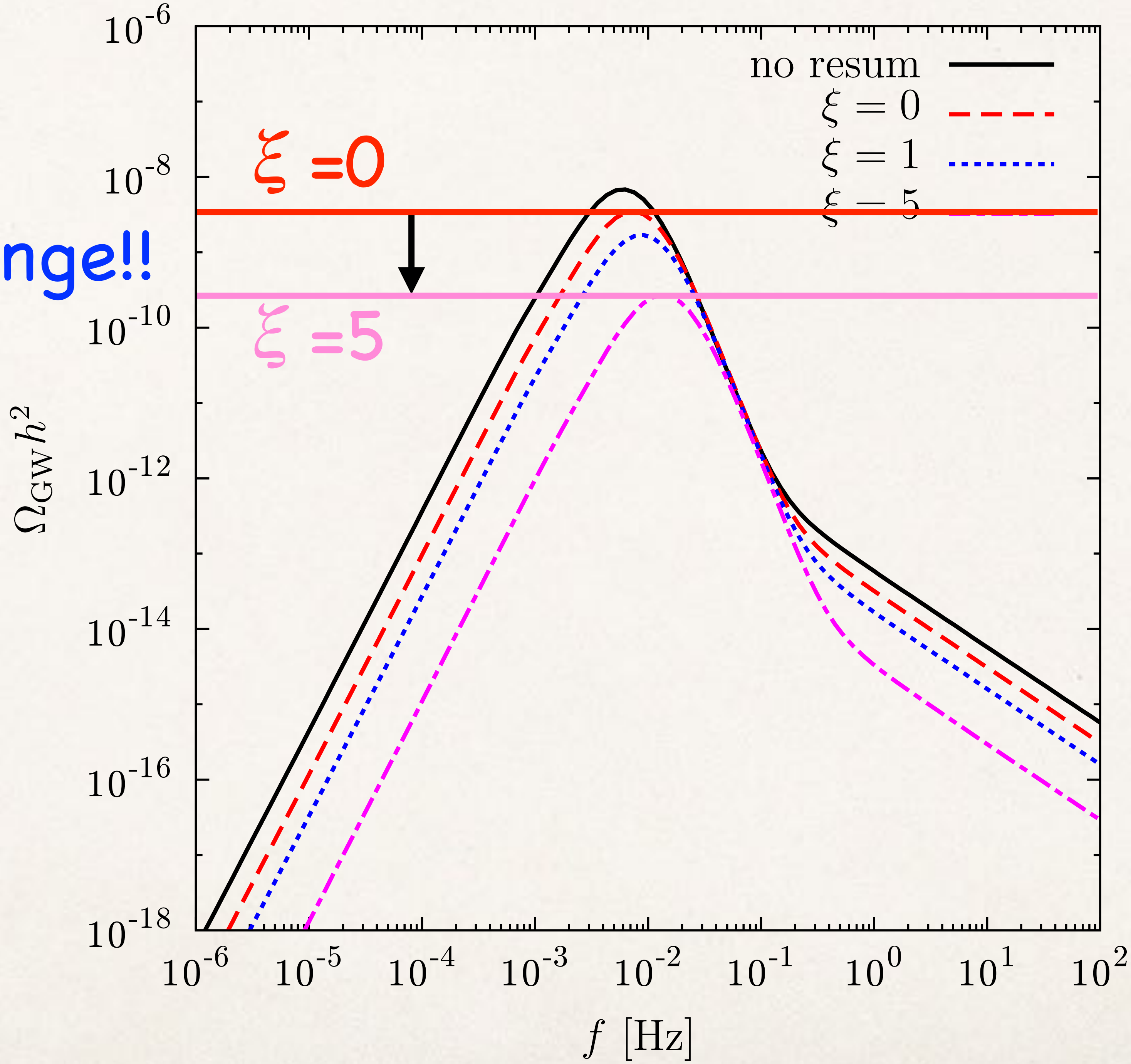


Impact of ξ on gravitational wave

~ 1 order change!!



ξ dependence of V_{eff} propagates to GW spectrum significantly!

Summary

- We have evaluated the gauge fixing parameter (ξ) dependence on GW from the 1st-order phase transitions.

- Effective potential is ξ dependent.

- Such ξ dependence can propagate to nucleation temperature and eventually gravitational waves.

  Ω_{GW} can change O(1) in magnitude varying ξ =0-5.

- Gauge-inv. method with consistent thermal resummation is necessary to get reliable results.

Backup

1-loop effective potential T≠0

Using a high-T expansion, one gets

$$
\begin{aligned}
&V_1(\varphi,\xi)+V_1(\varphi,\xi;T)\\
&=\frac{T^2}{24}(\bar m_h^2+\bar m_G^2+3\bar m_A^2)-\frac{T}{12\pi}\left[(\bar m_h^2)^{3/2}+(\bar m_G^2+\xi\bar m_A^2)^{3/2}+(3-\xi^{3/2})(\bar m_A^2)^{3/2}\right]\\
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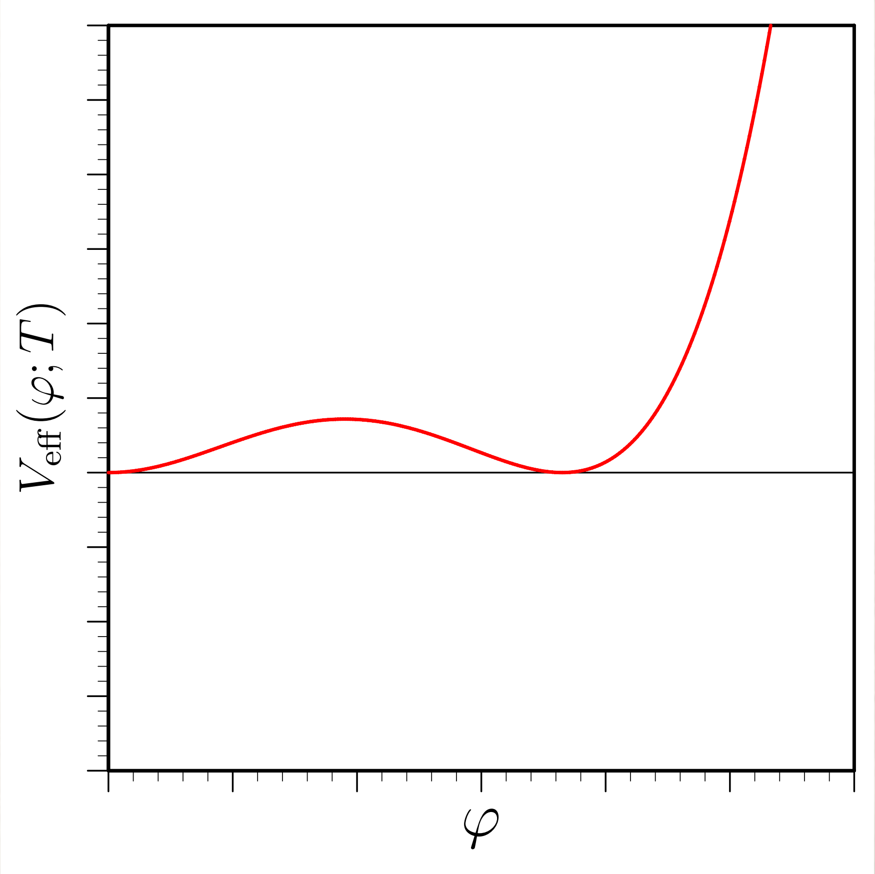

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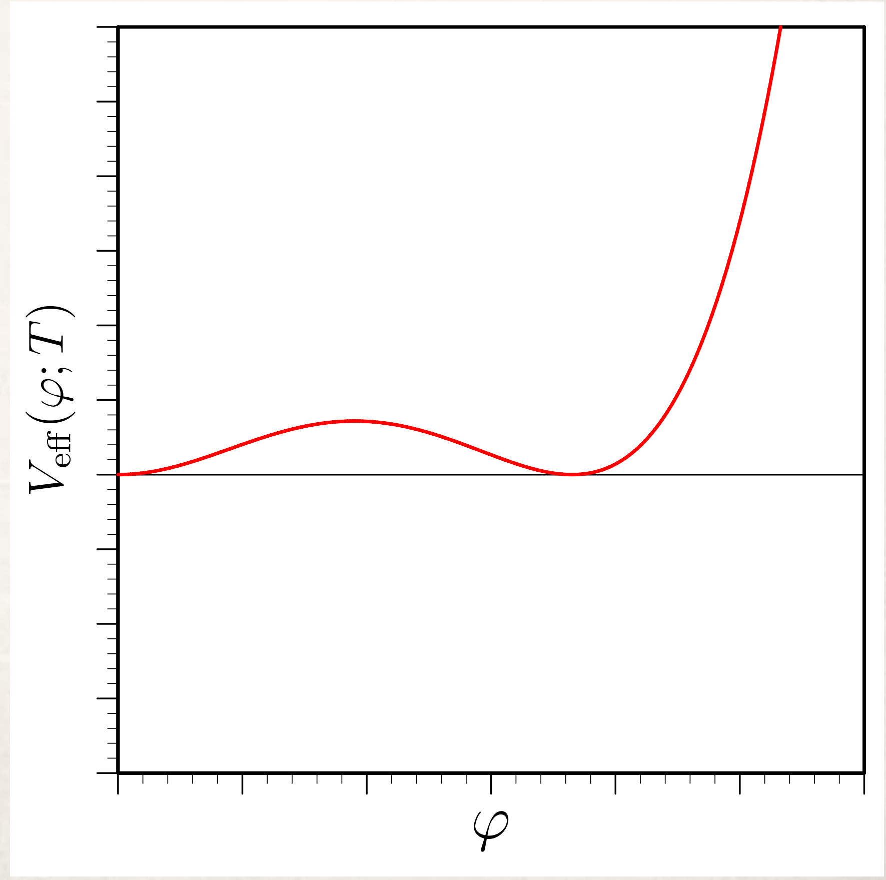

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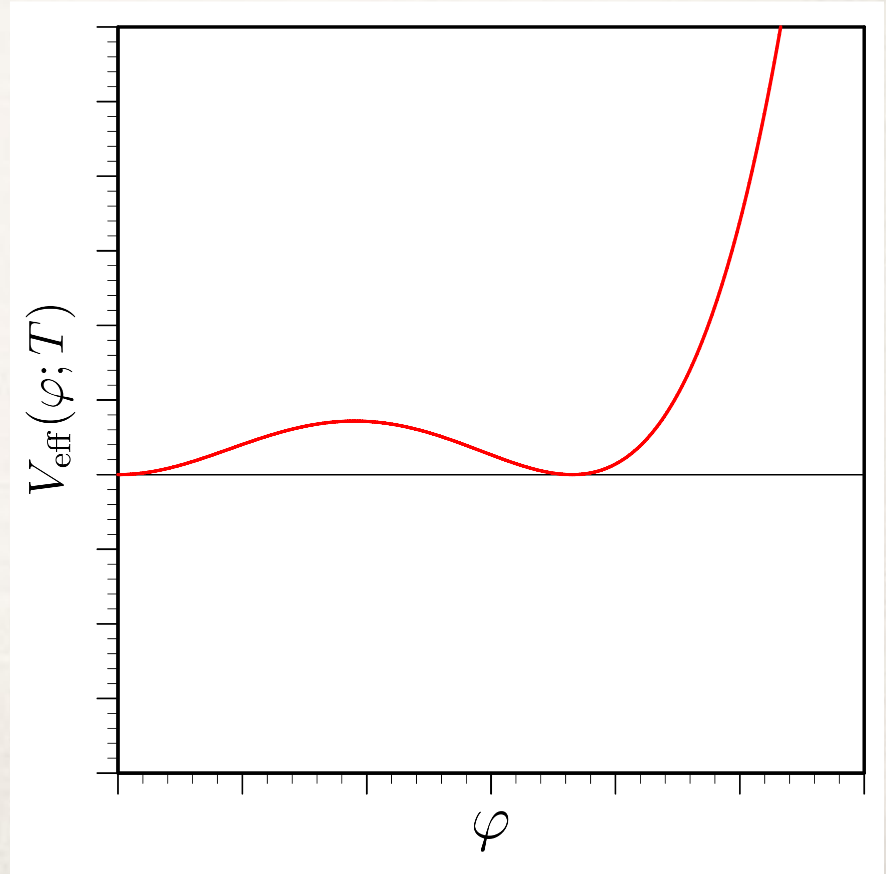

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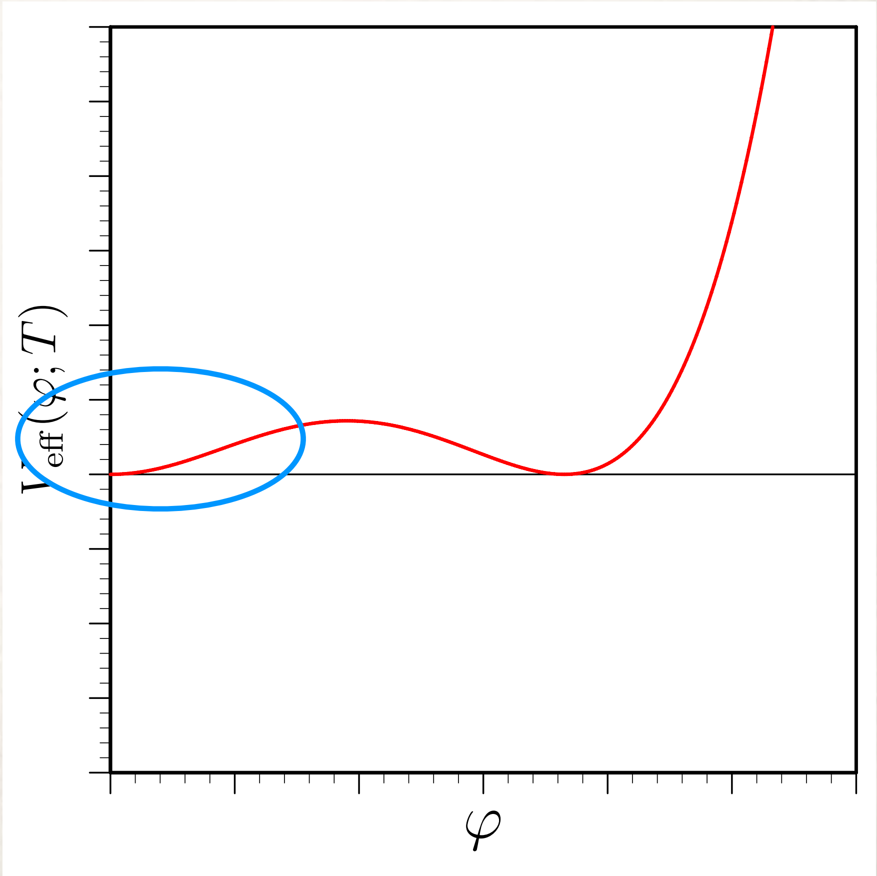

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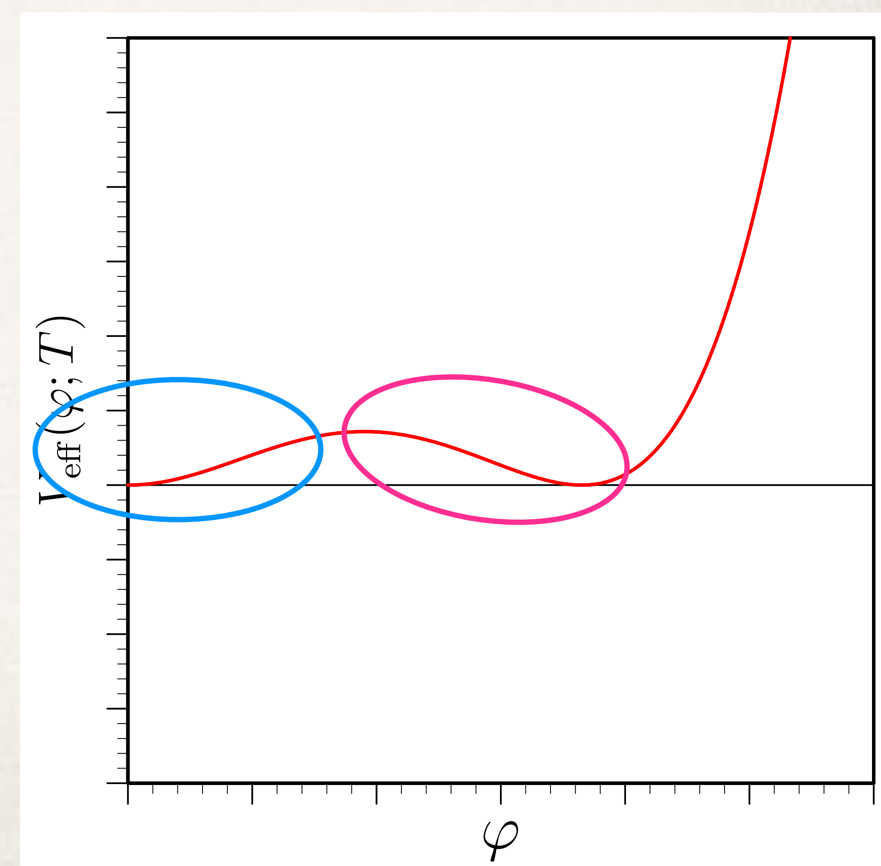

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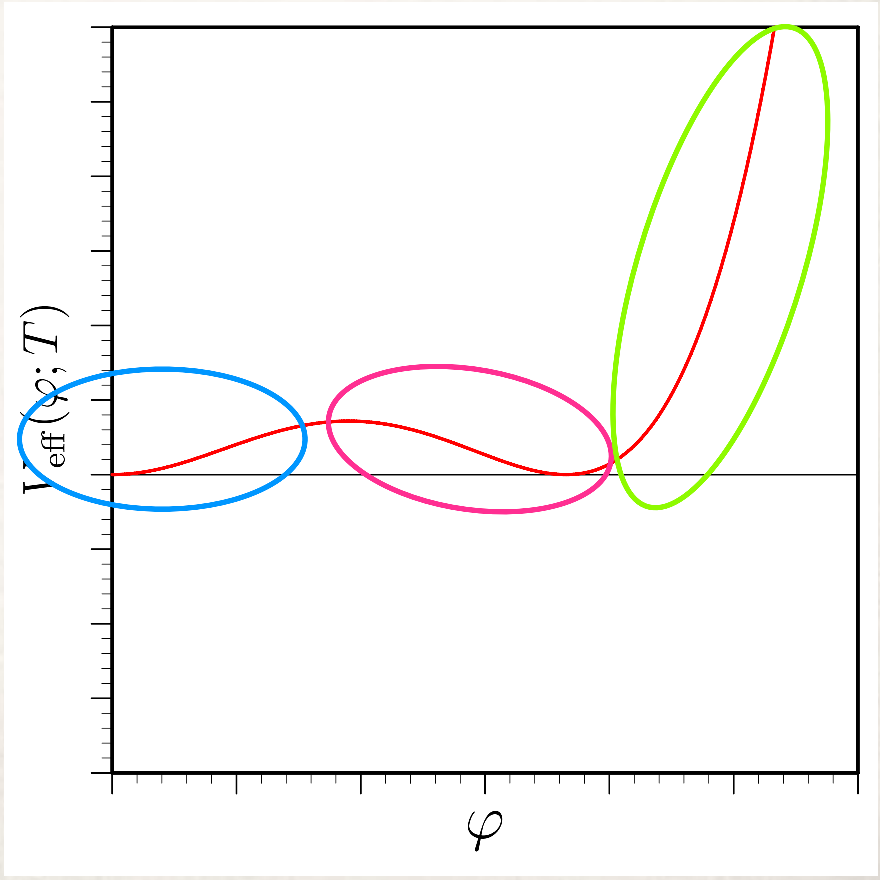

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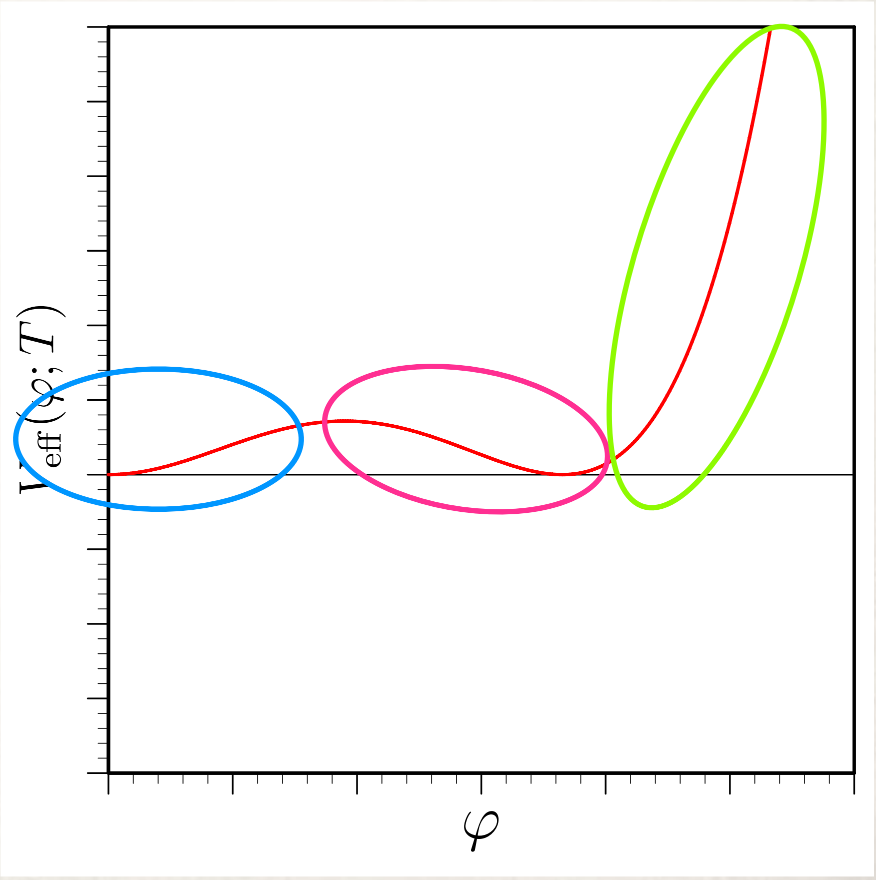


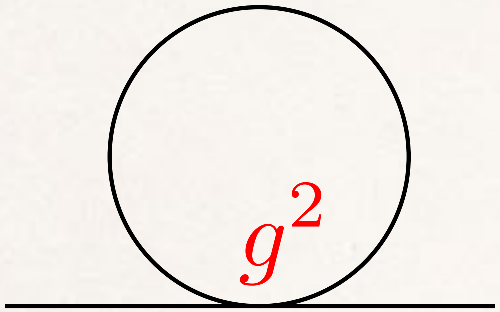
$$\frac{v_C}{T_C}=\frac{2E}{\lambda_T}$$

gauge dependent!!

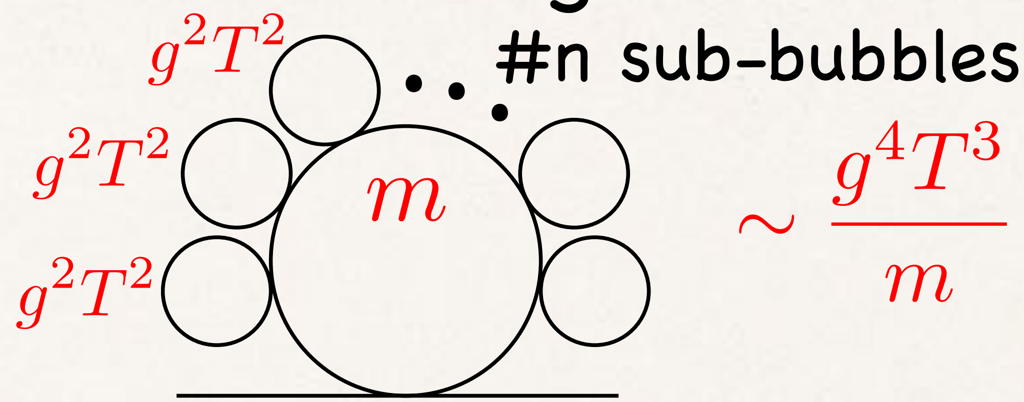
Thermal resummation

[Many refs: see, e.g., Parwani (92), Buchmüller et al (93), Chiku, Hatsuda (98), etc.]

Perturbative expansion gets worse at high T.



$\sim g^2T^2$



$$\sim \frac{g^4T^3}{m}\left(\frac{g^2T^2}{m^2}\right)^{n-1}$$

Dominant thermal terms are added and subtracted in the Lagrangian:

$$\mathcal{L}_B = \mathcal{L}_R + \mathcal{L}_{\rm CT} \to \left[\mathcal{L}_R + \Delta m_S^2|S|^2 + \frac{1}{2}\Delta m_L^2 Z'^\mu L_{\mu\nu}(i\partial)Z'^\nu + \frac{1}{2}\Delta m_T^2 Z'^\mu T_{\mu\nu}(i\partial)Z'^\nu\right]$$
$$+ \left[\mathcal{L}_{\rm CT} - \Delta m_S^2|S|^2 - \frac{1}{2}\Delta m_L^2 Z'^\mu L_{\mu\nu}(i\partial)Z'^\nu - \frac{1}{2}\Delta m_T^2 Z'^\mu T_{\mu\nu}(i\partial)Z'^\nu\right]$$

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[N.B.] Resummed Lagrangian preserves the gauge invariance.

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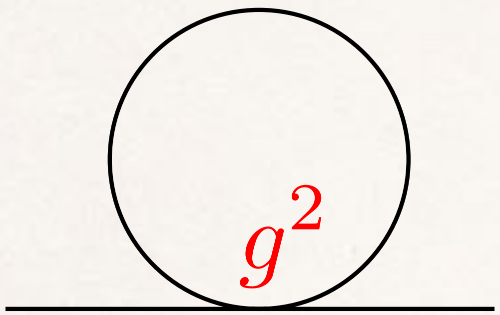

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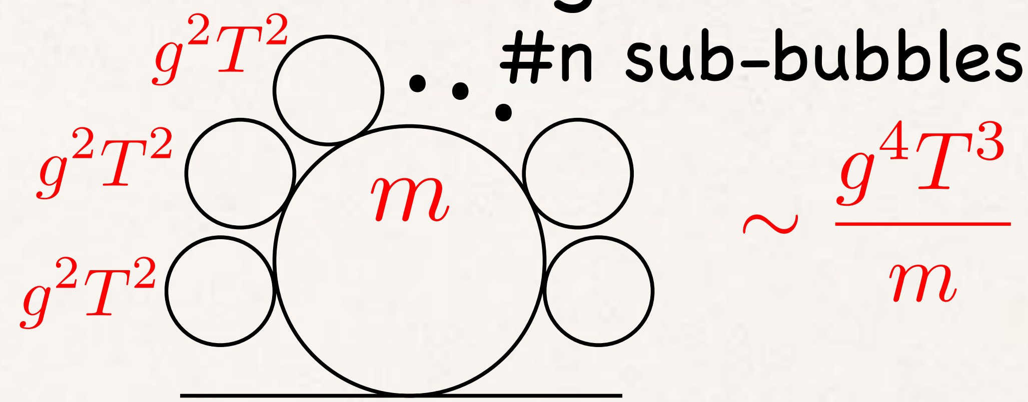

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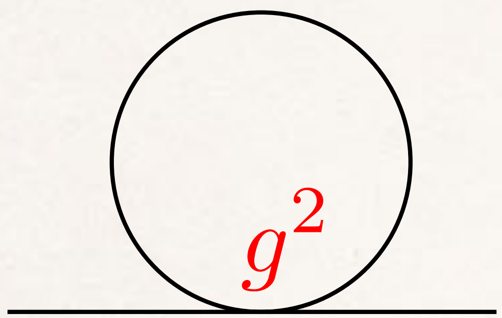

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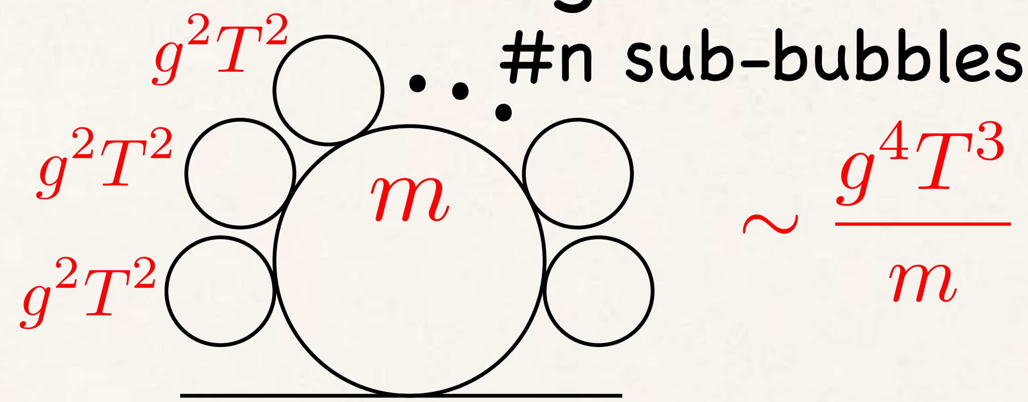

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new counterterm

where $T_{00} = T_{0i} = T_{i0} = 0\ , \quad T_{ij} = g_{ij} - \dfrac{k_i k_j}{-\boldsymbol{k}^2}\ ,$

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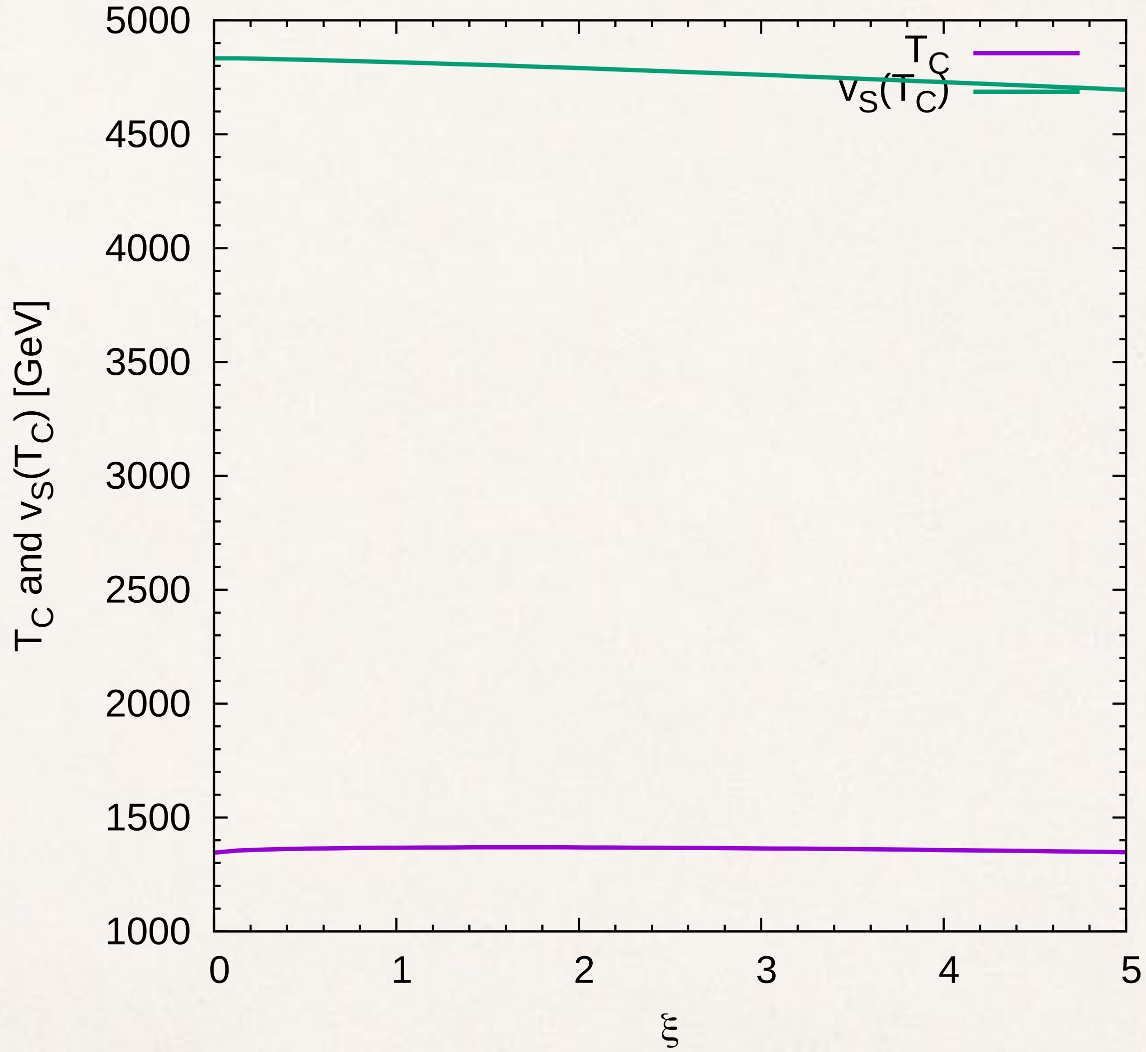
Impact of ξ on v/T

e.g., scale-inv. $U(1)_{B-L}$ model [Cheng-Wei Chiang, E.S., 1707.06765 (PLB)]

$$Q'_S = 2,\ \alpha' = g'^2/4\pi = 0.015,\ m_{Z'} = 4.5\ \text{TeV and } m_{\nu_{R1,2,3}} = 1.0\ \text{TeV}.$$



	no resum	$\xi = 0$	$\xi = 1$	$\xi = 5$
$\dfrac{v_S(T_C)}{T_C}$	$\dfrac{4.851}{1.321} = 3.67$	$\dfrac{4.833}{1.346} = 3.59$	$\dfrac{4.816}{1.368} = 3.52$	$\dfrac{4.695}{1.348} = 3.48$

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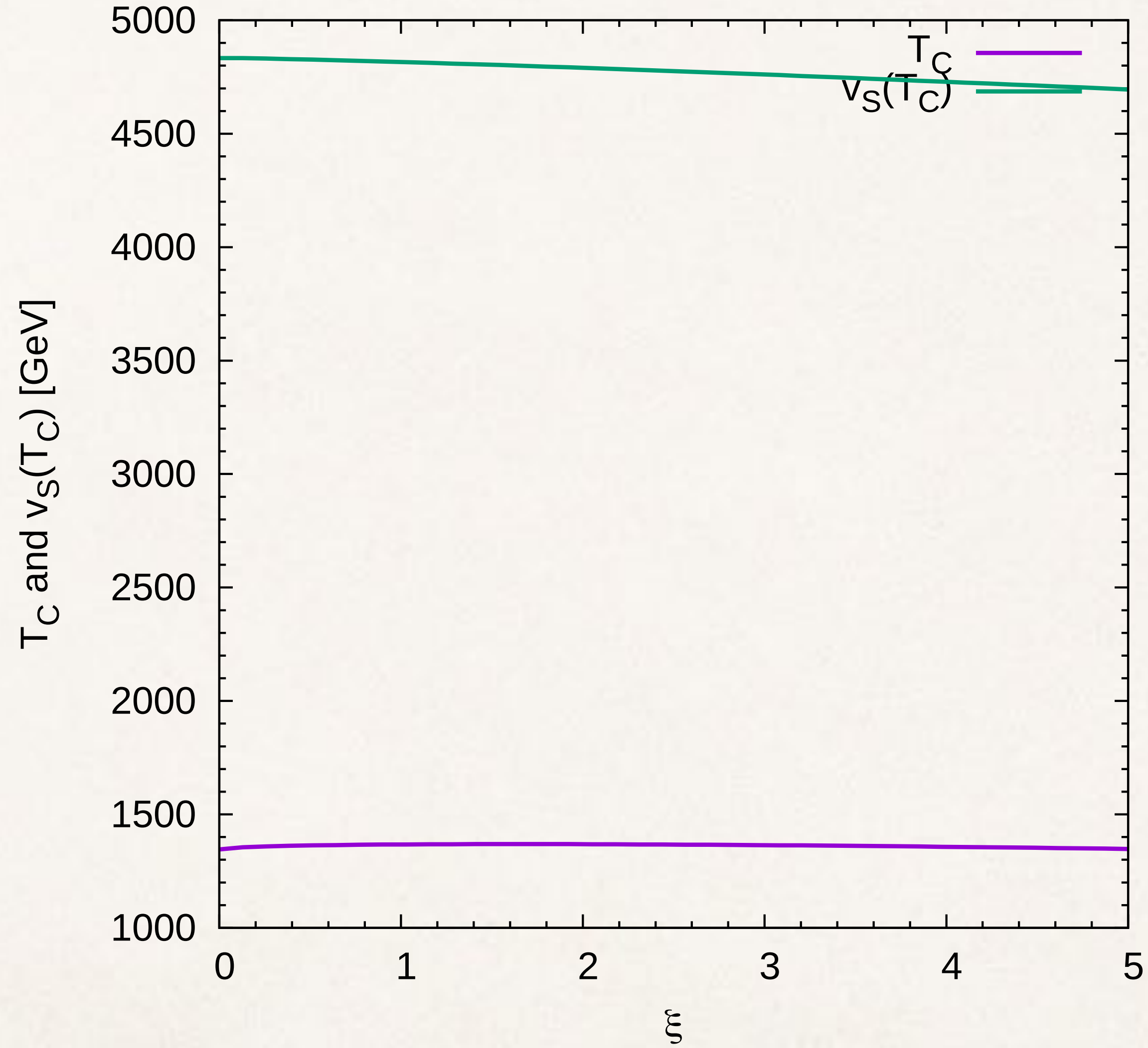

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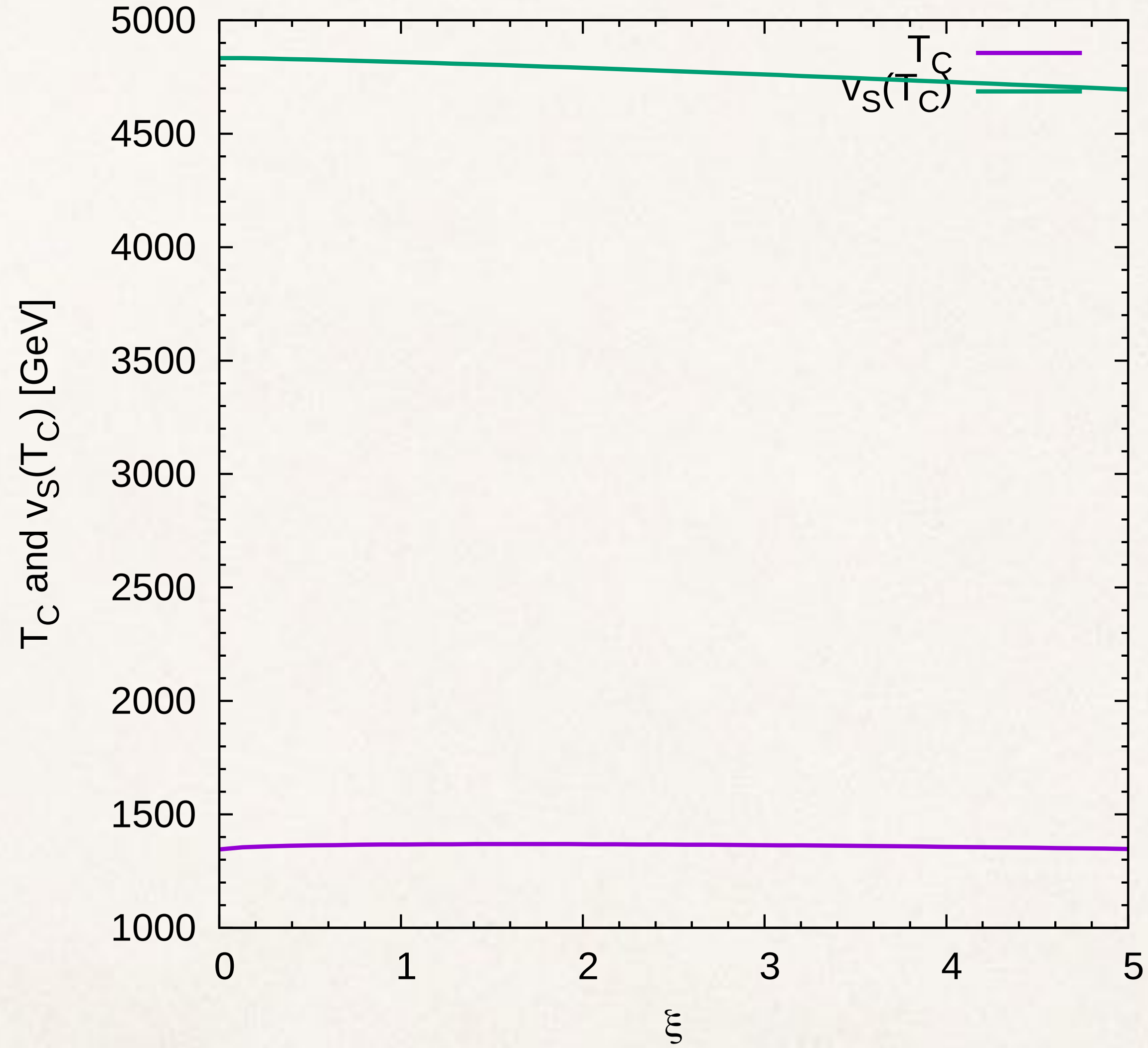

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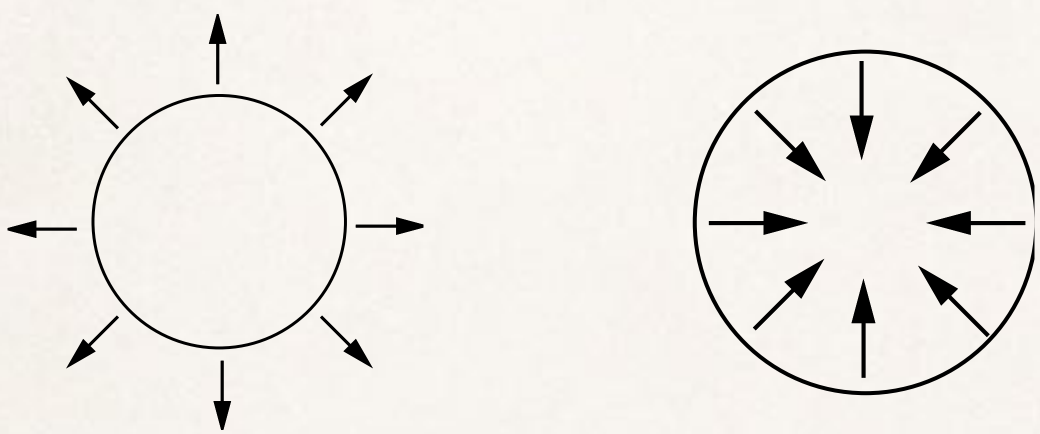
Onset of PT

- T_C is not onset of the PT.

- Nucleation starts somewhat below T_C.

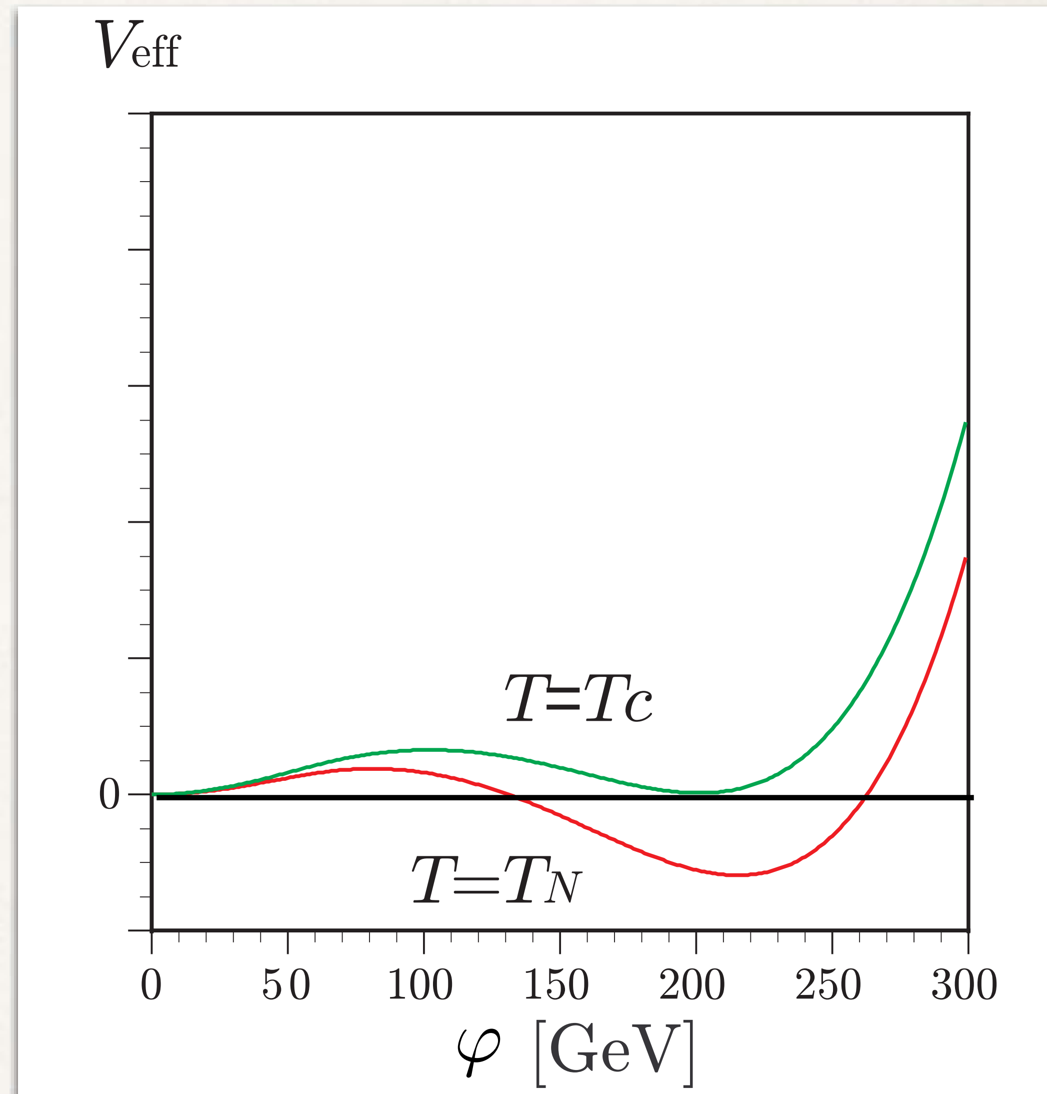
"Not all bubbles can grow"



expand? or shrink?

volume energy vs. surface energy

$\propto$(radius)^3 $\propto$(radius)^2



There is a critical value of radius -> critical bubble

Nucleation temperature

- Nucleation rate per unit time per unit volume

$$\Gamma_N(T) \simeq T^4 \left(\frac{S_3(T)}{2\pi T} \right)^{3/2} e^{-S_3(T)/T}$$

[A.D. Linde, NPB216 ('82) 421]

$S_3(T)$: energy of the critical bubble at T

- Definition of nucleation temperature (T_N)

horizon scale $\simeq H(T)^{-1}$

$$\Gamma_N(T_N) H(T_N)^{-3} = H(T_N)$$

$$\frac{S_3(T_N)}{T_N} - \frac{3}{2} \ln \left(\frac{S_3(T_N)}{T_N} \right) = 152.59 - 2 \ln g_*(T_N) - 4 \ln \left(\frac{T_N}{100 \text{ GeV}} \right)$$

Roughly, $S_3(T)/T \lesssim 150$ is needed for the PT.